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The Influence of Magnetohydrodynamic Flow and Slip Condition on Generalized Burgers' Fluid with Fractional Derivative

Amaal Mohi Nassief [1*] *Mohammed Ali Murad* [2]

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Abstract:

This paper investigates the effect of magnetohydrodynamic (MHD) of an incompressible generalized burgers' fluid including a gradient constant pressure and an exponentially accelerate plate where no slip hypothesis between the burgers' fluid and an exponential plate is no longer valid. The constitutive relationship can establish of the fluid model process by fractional calculus, by using Laplace and Finite Fourier sine transforms. We obtain a solution for shear stress and velocity distribution. Furthermore, 3D figures are drawn to exhibit the effect of magneto hydrodynamic and different parameters for the velocity distribution.

Key words: Burgers' fluid, Fox H-function, Laplace transform, Slip condition.

Introduction:

Recently, non-Newtonian fluids theory has many interesting theories so classical Newtonian fluids cannot characterize the most of fluids in technology and industry. There is not any a linear relationship between the rate of strain and the stress at a point in fluid mathematically. These fluids have been modeled in a number of different types with their constitutive equations changing greatly in complexness. Between the non- Newtonian liquids there is viscoelastic fluids (have both viscous and elastic identify) which are most commonly used. Consequently ,several equations of viscoelastic fluid are suggested by Oldroyd _ B fluid models(1-3) Maxwell fluids(4) and Burgers' fluids model(5).

The derivative fractional succeeded with the constitutive equations of viscoelastic fluids used to describe viscous properties, with are modified by substituting the derivative time of an integer derivate fractional with constitutive equations by Riemann -Liouville from operator fractional(6, 7).

Zheng et al (8),discussed 3D flow between two side walls perpendicular that was generalized in oldroyd_ B fluid because of a constant pressure gradient. Fetecua.et.al (9, 10).discussed exact solution for oldroyd_B fluid between the two side wall that perpendicular to the plat.

[1] Department of Mathematics, College of Science, University of Diyala, Diyala, Iraq
[2] Department of Mathematics, College of basic education, University of Diyala, Diyala, Iraq
[*]Corresponding author: amalmuhi@sciences.uodiyala.edu.iq
[*]ORCID ID: 0000-0003-0279-0758

Y. Liu et al (11), has studied the effect of radiation with heat transfer and magnetohydrodynamic flow by burgers' fluids because of the exponential accelerating plate. Ebaid (12) has studied the Effect of MHD on peristaltic transport of a Newtonian fluid in an asymmetric channel with slip condition .Yaqing Liu .et al(13) investigated the magnetohydrodynamic which generalized Maxwell fluid that induced of moving plate and effects of second-order slip. Ghada et al, (14) investigated the magneto hydrodynamic flow of burgers' fluid by flowing and accelerating plate under the influence of the gradient pressure .Shihao et al(15), investigated effect of slip on 3D flow on fluid Burger between two side of wall generated by accelerated plate and a constant gradient pressure , Zheng et al(16) investigated studied slip effect on magnetohydrodynamic which be an Oldroyd_B fluid beginning by an accelerated plate.

In this research, we discuss the Influence of magneto hydrodynamic (MHD) on 3D Flow for Burgers' Fluid with Slip Condition between the two side of the walls that generated by a constant pressure gradient and exponential accelerated plate. The best solution of the velocity distribution and shear stress are acquired by using the Fourier Sine and Laplace transformations.

The description of the problem and its solution

Suppose that the generalized burgers' fluid with fractional derivative between the two sides of wall which occupy the complete space over the plate that is perpendicular to the side of wall. The fluid starts to move because of the exponentially accelerated plate with a movement of this velocity *Exp(-t)* and a pressure constant *B* and by the presence of the slip condition .The related boundary condition and initial condition are as follows:

$$\mathrm{u}(y,z,0)=\partial_t\,\mathrm{u}(y,z,0)=\frac{\partial^2\,\mathrm{u}(y,z,0)}{\partial t^2}=0\,,\quad (0\le z\le d\ and\ y>0)\qquad \dots\ (1)$$

$$\mathrm{u}(0,z,t)=Exp(-t)\quad+\varphi\,\partial_y\mathrm{u}(0,z,t),\quad (0\le z\le d\ \text{and}\ t\ge 0)\qquad \dots(2)$$

$$\mathrm{u}(y,z,t)=\partial_y\mathrm{u}(y,z,t)=0\ ,(t\ge 0\ and\ (z=d,0)\quad ,y\ge 0)\qquad \dots(3)$$

$$\partial_y\mathrm{u}(y,z,t)\to 0\,,(t\ge 0\ and\ y\to\infty,\ 0<z<d\,)\qquad \dots\ (4)$$

Where (φ)represents the slip coefficient, (*d*) represents the distance between the two side walls**.**

The equalization of an incompressible and generalized burger's fluid are presented by (6):

$$T=-pI+S,$$

$$S\left(1+\lambda_1^\alpha\frac{D^\alpha}{D_t}+\lambda_1^\alpha\frac{D^{2\alpha}}{D_t}\right)=\mu\left(1+\lambda_3^\beta\frac{D^\beta}{D_t}\right)A\ \dots(5)$$

Where *-pI* represents the Indeterminate Spherical Stress , *T* represents the Cauchy Stress Tensor, *S* represents the extra stress tensor , λ_2 a new material constant of burgers' fluid , λ_1 represents the relaxation time and λ_3 represents the retardation times , μ represents viscosity coefficient, $A=L+(L)^T$ is the first Rivlin Ericksen Tensor , $L=\mathrm{grad}\,V$ is the velocity gradient, α,β are the fractional calculus parameters, such that $0\le\alpha\le\beta\le 1$ and $\frac{D^\alpha}{D_t}$ represent the upper convected time derivative define by (6) :

$$\frac{D^\alpha}{D_t}S=D_t^\alpha S+(V.\nabla)S-L.S-S.\mathrm{L}^T,\qquad \dots\ (6)$$

$$\frac{D^\beta}{D_t}A=D_t^\beta A+L.A-L.A-A.L^T\qquad \dots\ (7)$$

Where $\frac{D^{2\alpha}}{D_t}S=\frac{D^\alpha}{D_t}(\frac{D^\alpha}{D_t}S\ )$.

Where ∇ is the gradient operator and *V* represent the velocity vector ,D_t^q denoted the fractional operator is defined by (7):

$$\frac{D^q}{D_t}f(t)=\frac{1}{\mathrm{Gamm}(1-q)}\frac{d}{\mathrm{d}t}\int_0^t\frac{f(\tau)}{(t-\tau)^q}\qquad d\tau\,,\quad 0\le q<1\qquad \dots\ (8)$$

Here Gamm(.) denotes the Gamma function.

We assume the stress and velocity of the form

$$S=S(y,z,t),\ V=\mathrm{u}(y,z,t)\hat{\imath}\qquad \dots\ (9)$$

where $\hat{\imath}$ is the unit vector along the x- coordinate direction and using the initial condition $S(y,z,0)=0$, we find

$$(1+\lambda_1^\alpha D_t^\alpha\ +\lambda_2^\alpha\ D_t^{2\alpha}\ )\tau 1=\mu\ \left(1+\lambda_3^\beta\ D_t^\beta\right)\ \partial_y u(y,z,t)(10)$$

$$(1+\lambda_1^\alpha D_t^\alpha\ +\lambda_2^\alpha\ D_t^{2\alpha}\ )\tau 2=\mu\ \left(1+\lambda_3^\beta\ D_t^\beta\right)\ \partial_z u(y,z,t)\ (11)$$

And, suppose that a burger fluid is penetrated by a magnetic field B_0 that is applied parallel to the y-axis while the magnetohydrodynamic ignored by taking a fewest magnetic Reynolds number. Hence, the MHD body force caused by the external magnetic field takes the form σB_0^2 ,in which B_0 is the magnitude and σ represent the electrical conductivity of the fluid then the motion equation yield the following

$$\rho\,\partial_t\mathrm{u}(y,z,t)=\frac{-\partial P}{\partial x}+\ \partial_y\tau 1+\ \partial_z\tau 2-\sigma B_0^2\mathrm{u}\qquad \dots\ (12)$$

Where $\frac{\partial P}{\partial x}$ represents the pressure gradient along x-axis and $S_{xy}=\tau 1, S_{xz}=\tau 2$ are the tangential stresses different from zero.

Then from Equation (10), Equation (11) and Equation (12), we obtain the governing equation for the generalized fractional Burgers' fluid

$$(1+\lambda_1^\alpha\ D_t^\alpha+\lambda_2^\alpha\ D_t^{2\alpha})\frac{\partial\mathrm{u}(y,z,t)}{\partial t}=B(1+\lambda_1^\alpha\ D_t^\alpha+\lambda_2^\alpha\ D_t^{2\alpha})+\nu\left(1+\lambda_3^\beta D_t^\beta\right)\left(\frac{\partial^2\mathrm{u}(y,z,t)}{\partial y^2}+\frac{\partial^2\mathrm{u}(y,z,t)}{\partial z^2}\right)-\ \mathrm{u}(y,z,t)(1+\lambda_1^\alpha\ D_t^\alpha+\lambda_2^\alpha\ D_t^{2\alpha}\ )\ \mathrm{N}\qquad \dots(13)$$

Where $\nu=\frac{\mu}{\rho}$ represents the kinematic viscosity, $B=\frac{-1}{\rho}\frac{\partial P}{\partial x}$ represents the constant pressure gradient in the x-axis direction and $bN=\frac{\sigma B_0^2}{\rho}$.

We get the solution of velocity distribution by using the Finite Fourier Sine and Laplace transformations with series fractional derivative (6).

Now, by multiplying two sides of Equation (13) through $\sin(\frac{n\pi z}{d})$ and integrate w.r.t z from 0 to d, we obtain the equation:

$$(1+\lambda_1^\alpha D_t^\alpha+\lambda_2^\alpha D_t^{2\alpha})\partial_t\mathrm{u_n}(y,n,t)=B\frac{d}{n\pi}(1-(-1)^n)\left(1+\lambda_1^\alpha\frac{t^{-\alpha}}{\Gamma(1-\alpha)}+\lambda_2^\alpha\frac{t^{-2\alpha}}{\Gamma(1-2\alpha)}\right)+\nu\left(1+\lambda_3^\beta D_t^\beta\right)\frac{\partial^2\mathrm{u_n}(y,n,t)}{\partial y^2}-N(1+\lambda_1^\alpha D_t^\alpha+\lambda_2^\alpha D_t^{2\alpha})\mathrm{u_n}(y,n,t)-\nu\left(\frac{n\pi}{d}\right)^2\left(1+\lambda_3^\beta D_t^\beta\right)\mathrm{u_n}(y,n,t)\qquad \dots\ (14)$$

Using the Laplace transform of Equation (14), we obtain it

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$$\frac{\partial^2 u_{pn}(y,n,p)}{\partial y^2} - \left(\delta^2 + \frac{(p+N)\left(1+\lambda_1^{\alpha} p^{\alpha} + \lambda_2^{\alpha} p^{2\alpha}\right)}{\nu\left(1+\lambda_3^{\beta} p^{\beta}\right)}\right) u_{pn}(y,n,t) + \frac{B}{\delta\nu}\,(1-(-1)^n)\,\frac{\left(1+\lambda_1^{\alpha} p^{\alpha} + \lambda_2^{\alpha} p^{2\alpha}\right)}{p\left(1+\lambda_3^{\beta} p^{\beta}\right)} = 0 \quad \ldots \quad (15)$$

Where $\frac{n\pi}{d} = \delta$,the Laplace transform principle (6) is

$$\mathrm{u_p}(y,z,p) = \int_0^{\infty} u(y,z,t)\ Exp(-pt)dt \quad ,p>0 \quad \ldots (16)$$

We obtain the Equation (17) by utilize the ordinary differential equations to Equation (15)

$$u_{pn}(y,n,p)\left(\frac{\frac{(-(-1)^n+1)}{\delta(p+1)} - \frac{B(-(-1)^n+1)}{\delta p\left(\nu \mathrm{H}\delta^2 + (p+\mathrm{N})\right)}}{1+\varphi\sqrt{\delta^2+\frac{p+\mathrm{N}}{\nu H}}}\right).Exp\left(-\sqrt{\delta^2+\frac{p+\mathrm{N}}{\nu H}}y\right)\frac{B(-(-1)^n+1)}{\delta p\left(\nu \mathrm{H}\delta^2+(p+\mathrm{N})\right)} \ldots (17)$$

Where $\mathrm{H} = \frac{1+\lambda_3^{\beta} p^{\beta}}{1+\lambda_1^{\alpha} p^{\alpha} + \lambda_2^{\alpha} p^{2\alpha}}$

Here in organize to avert the lengthy procedure of residues integral and contour integral, so we rewriting the Equation (17) as

$$\mathrm{u_{pn}}(y,n,p) = \mathrm{u_{pn1}}(y,n,p) + \mathrm{u_{pn2}}(y,n,p) + \mathrm{u_{pn3}}(y,n,p) \quad \ldots \quad (18)$$

Writing $\mathrm{u_{pn1}}(y,n,p)$in series form as (using $\frac{1}{s+a} = \sum_{i=0}^{\infty}(-1)^i\ \frac{s^i}{a^{i+1}}$)

$$\frac{1}{p+1} = \sum_{i=0}^{\infty}(-1)^i\,p^i\ ,\quad \frac{1}{1+\varphi\sqrt{\delta^2+\frac{p+\mathrm{N}}{\nu H}}} = \sum_{r=0}^{\infty}\frac{(-1)^r}{\varphi^{r+1}}\left(\delta^2+\frac{p+\mathrm{N}}{\nu H}\right)^{\frac{-r-1}{2}} \quad and$$

$$Exp\left(-\sqrt{\delta^2+\frac{p+\mathrm{N}}{\nu H}}y\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\,y^k\left(\delta^2+\frac{p+\mathrm{N}}{\nu H}\right)^k$$

By merging equations above and doing some procedure using a source (18) we get the following equation

$$\mathrm{u_{pn1}}(y,n,p) = (1-(-1)^n)\sum_{l=0}^{\infty}\sum_{\zeta=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{i=0}^{\infty}\frac{(-1)^{k+\zeta+\omega+i+r+l+j}}{k!\,\zeta!\,l!\,\omega!\,j!}y^k\left(\frac{1}{\varphi}\right)^{r+1}\delta^{2\zeta-1}$$
$$\nu^{\zeta+q}N^l\frac{\Gamma(l+\zeta+q)\Gamma(\zeta-q)\Gamma(\omega-q-\zeta)\Gamma(j+\zeta+q)}{\Gamma(\zeta+q)\Gamma(-q)\Gamma(-q-\zeta)\Gamma(\zeta+q)}\sum_{o=0}^{j}\frac{j!}{o!\,(j-o)!}\left(\frac{1}{\lambda_1^{\alpha}}\right)^{\zeta+q+j}\lambda_3^{\beta(\zeta+q-\omega)}(\lambda_2^{\alpha o})$$
$$\frac{1}{p^{\zeta+q+l-\beta(\zeta+q-\omega)+\alpha(\zeta+q+j)-2\alpha o-i}}$$

By the same method we get the following equations:

$$\mathrm{u_{pn2}}(y,n,p) = -B(1-(-1)^n)\sum_{j=0}^{\infty}\sum_{r=0}^{\infty}\sum_{l=0}^{\infty}\sum_{\zeta=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{k+\zeta+\omega+r+l+j}}{k!\,\zeta!\,l!\,\omega!\,j!}y^k\left(\frac{1}{\varphi}\right)^{r+1}\delta^{2\zeta-1}\nu^{\zeta-q}$$
$$\sum_{o=0}^{j}\frac{j!}{o!\,(j-o)!}N^l\lambda_3^{\beta(-\omega-q+\zeta)}\lambda_1^{\alpha(-j-\zeta+q)}\lambda_2^{\alpha o}\frac{\Gamma(\zeta+1-q)}{\Gamma(1-q)}\frac{\Gamma(l+\zeta+q-1)\Gamma(\omega+q-\zeta)\Gamma(j+\zeta-q)}{\Gamma(\zeta+q-1)\Gamma(q-\zeta)\Gamma(\zeta-q)}$$
$$\frac{1}{p^{\zeta+q+l-\beta(\zeta-q-\omega)+\alpha(\zeta-q+j)-2\alpha o}}$$

$$u_{pn3}(y,n,p) = B(1-(-1)^n)\sum_{k=0}^{\infty}\sum_{\zeta=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^{k+\zeta+\omega+l}}{\zeta!\,l!\,\omega!}\delta^{-2(k+1)-1}\nu^{-k-1}N^{-\zeta}\frac{\Gamma(\zeta-k)}{\Gamma(-k)}$$

$$\frac{\Gamma(l-k-1)\Gamma(\omega+k+1)}{\Gamma(-k-1)\Gamma(k+1)}\sum_{o=0}^{l}\frac{l!}{o!\,(l-o)!}\lambda_3^{\beta(-\omega-k-1)}\lambda_1^{\alpha(k+1-l)}\lambda_2^{\alpha o}\frac{1}{p^{1-k-\zeta+\beta(1+k+\omega)+\alpha(-k-1+l)-2\alpha o}}$$

Where $q=\frac{k-r-1}{2}$

We use the Inverse Laplace transform method for Equation (18), we get

$$u_n(y,n,t) = u_{n1}(y,n,t) + u_{n2}(y,n,t) + u_{n3}(y,n,t) \quad \dots (19)$$

By applying invers Laplace $to\ l^{-1}(u_{pn1}(y,n,p)) = u_{n1}(y,n,t)$ and using (6)

$$l^{-1}\left(\frac{1}{p^{\zeta+q+l-\beta(\zeta+q-\omega)+\alpha(\zeta+q+j)-2\alpha o-i}}\right)=\frac{t^{\zeta+q+l-\beta(\zeta+q-\omega)+\alpha(\zeta+q+j)-2\alpha o-i-1}}{\Gamma(\zeta+q+l-\beta(\zeta+q-\omega)+\alpha(\zeta+q+j)-2\alpha o-i)}$$

We get,

$$u_{n1}(y,n,t) = (1-(-1)^n)\sum_{l=0}^{\infty}\sum_{r=0}^{\infty}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{\zeta=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^{k+\zeta+\omega+i+r+l+j}}{l!\,\zeta!\,k!\,\omega!\,j!}y^k\left(\frac{1}{\varphi}\right)^{r+1}\delta^{2\zeta-1}\nu^{\zeta+q}N^l$$

$$\frac{\Gamma(l+\zeta+q)\Gamma(\zeta-q)\Gamma(\omega-q-\zeta)\Gamma(j+\zeta+q)}{\Gamma(\zeta+q)\Gamma(-q)\Gamma(-q-\zeta)\Gamma(\zeta+q)}\sum_{o=0}^{j}\frac{j!}{o!\,(j-o)!}\left(\frac{1}{\lambda_1^{\alpha}}\right)^{\zeta+j+q}\lambda_3^{\beta(\zeta+q-\omega)}(\lambda_2^{\alpha o})$$

$$\frac{t^{\zeta+q+l-\beta(\zeta+q-\omega)+\alpha(\zeta+q+j)-2\alpha o-i-1}}{\Gamma(\zeta+q+l-\beta(\zeta+q-\omega)+\alpha(\zeta+q+j)-2\alpha o-i)}$$

And in the same method, we get the following equations

$$u_{n2}(y,n,t) = -B(1-(-1)^n)\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\sum_{l=0}^{\infty}\sum_{j=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{\zeta=0}^{\infty}\frac{(-1)^{l+\zeta+\omega+r+k+j}}{k!\,\zeta!\,l!\,\omega!\,j!}y^k\left(\frac{1}{\varphi}\right)^{r+1}\delta^{2\zeta-1}\nu^{\zeta-q}$$

$$\sum_{o=0}^{j}\frac{j\,!}{o!\,(j-o)!}N^l\lambda_3^{\beta(-\omega-q+\zeta)}\lambda_1^{\alpha(-j-\zeta+q)}\lambda_2^{\alpha o}\frac{\Gamma(\zeta+1-q)}{\Gamma(1-q)}\frac{\Gamma(l+\zeta+q-1)\Gamma(\omega+q-\zeta)\Gamma(j+\zeta-q)}{\Gamma(\zeta+q-1)\Gamma(q-\zeta)\Gamma(\zeta-q)}$$

$$\frac{t^{\zeta+q+l-\beta(\zeta-q)-\beta\omega+\alpha(\zeta-q+j)-2\alpha o-1}}{\Gamma(\zeta+q+l-\beta(\zeta-q)-\beta\omega+\alpha(\zeta-q+j)-2\alpha o)}$$

$$u_{n3}(y,n,t) = B(1-(-1)^n)\sum_{l=0}^{\infty}\sum_{\zeta=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{l+\zeta+k+\omega}}{\zeta!\,l!\,\omega!}\delta^{-2k-2}\nu^{-k-1}N^{-\zeta}\frac{\Gamma(-(k-\zeta))}{\Gamma(-k)}\frac{\Gamma(-(-l+k+1)}{\Gamma(-(k+1))}$$

$$\frac{\Gamma(k+\omega+1)}{\Gamma(k+1)}\sum_{o=0}^{l}\frac{l!}{o!\,(l-o)!}\lambda_3^{-\beta(\omega+1+k)}\frac{\lambda_1^{-\alpha(-k-1+l)}\lambda_2^{\alpha o}\,t^{-(\zeta+k)+\beta(k+\omega)+\beta-\alpha l+\alpha(k+1)-2\alpha o}}{\Gamma(1-(\zeta+k)+\beta(k+\omega)+\beta-\alpha l+\alpha(k+1)-2\alpha o)}$$

Applying the Inverse Finite Fourier Sine transform to Equation (19) and using source (17) ,we get the solution

$$u(y,z,t)=\frac{2}{d}\sum_{n=1}^{\infty}sin(\frac{n\pi z}{d})\big(u_{n1}(y,n,t)+u_{n2}(y,n,t)+u_{n3}(y,n,t)\big)$$

Where

$$u(y,z,t)=\frac{2}{d}\sum_{n=1}^{\infty}\sin(\frac{n\pi z}{d})(-(-1)^n+1)\sum_{j=0}^{\infty}\sum_{r=0}^{\infty}\sum_{\zeta=0}^{\infty}\sum_{i=0}^{\infty}\sum_{l=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{k+\zeta+\omega+i+r+l+j}}{k!\,\zeta!\,l!\,\omega!\,j!}y^k\left(\frac{1}{\varphi}\right)^{r+1}$$

$$\delta^{2\zeta-1}\nu^{\zeta+(q)}N^l\sum_{o=0}^{j}\frac{j!}{o!\,(j-o)!}\left(\frac{1}{\lambda_1^{\alpha}}\right)^{\zeta+q+j}\lambda_3^{\beta(\zeta+q-\omega)}(\lambda_2^{\alpha o})\frac{\Gamma(l+\zeta+q)\Gamma(\zeta-q)\Gamma(\omega-q-\zeta)}{\Gamma(\zeta+q)\Gamma(-q))\Gamma(-q-\zeta)}$$

$$\frac{\Gamma(j+\zeta+q)t^{\zeta+q+l-\beta(\zeta+q-\omega)+\alpha(\zeta+q+j)-2\alpha o-i-1}}{\Gamma(\zeta+q)\Gamma(\zeta+(q)+l-\beta(\zeta+q-\omega)+\alpha(\zeta+q+j)-2\alpha o-i)}$$

$$-\frac{2}{d}\sum_{n=1}^{\infty}\sin(\frac{n\pi z}{d})B(-(-1)^n+1)\sum_{\zeta=0}^{\infty}\sum_{r=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^{k+\zeta+\omega+r+l+j}}{k!\,\zeta!\,l!\,\omega!\,j!}$$

$$y^k\left(\frac{1}{\varphi}\right)^{r+1}\delta^{2\zeta-1}\nu^{\zeta-q}N^l\sum_{o=0}^{j}\frac{j!}{o!\,(j-o)!}\lambda_3^{\beta(-\omega-q+\zeta)}\lambda_1^{\alpha(-j-\zeta+q)}\lambda_2^{\alpha o}\frac{\Gamma(\zeta+1-q)}{\Gamma(1-q)}$$

$$\frac{\Gamma(l+\zeta+q-1)\Gamma(\omega+q-\zeta)\Gamma(j+\zeta-q)}{\Gamma(\zeta+q-1)\Gamma(q-\zeta)\Gamma(\zeta-q)}\frac{t^{\zeta+q+l-\beta(\zeta-q-\omega)+\alpha(\zeta-q+j)-2\alpha o-1}}{\Gamma(\zeta+q+l-\beta(\zeta-q-\omega)+\alpha(\zeta-q+j)-2\alpha o)}$$

$$+\frac{2}{d}\sum_{n=1}^{\infty}\sin(\frac{n\pi z}{d})B(-(-1)^n+1)\sum_{\zeta=0}^{\infty}\sum_{l=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{\zeta+l+\omega+k}}{\zeta!\,l!\,\omega!}\delta^{-2(k+1)-1}\nu^{-k-1}N^{-\zeta}$$

$$\sum_{o=0}^{l}\frac{l!}{o!\,(l-o)!}\lambda_1^{\alpha(k+1-l)}\lambda_2^{\alpha o}\lambda_3^{\beta(-\omega-k-1)}\frac{\Gamma(\zeta-\mathrm{k})}{\Gamma(-k)}\frac{\Gamma(-k-1+l)}{\Gamma(-(\mathrm{k}+1))}\frac{\Gamma(\omega+k+1)}{\Gamma(\mathrm{k}+1)}$$

$$\frac{t^{-(\zeta+k)+\beta k+\beta(\omega+1)-\alpha k+\alpha(1-l)-2\alpha o}}{\Gamma(1-(\zeta+k)+\beta k+\beta(\omega+1)-\alpha k+\alpha(1-l)-2\alpha o)}\quad\ldots(20)$$

We rewrite the Equation (20) by using the Fox H- Function ,we find

$$\mathrm{u(y,z,t)}=\frac{2}{d}\sum_{n=1}^{\infty}\sin(\frac{n\pi z}{d})(-(-1)^n+1)\sum_{\zeta=0}^{\infty}\sum_{l=0}^{\infty}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^{k+\zeta+i+r+l+j}}{k!\,l!\,\zeta!\,j!}y^k\left(\frac{1}{\varphi}\right)^{r+1}\delta^{2\zeta-1}$$

$$\nu^{\zeta+q}N^l\sum_{o=0}^{j}\frac{j!}{o!\,(j-o)!}\left(\frac{1}{\lambda_1^{\alpha}}\right)^{\zeta+q+j}\lambda_3^{\beta(\zeta+q)}(\lambda_2^{\alpha o})\,t^{\zeta+q+l-\beta(\zeta+q)+\alpha(\zeta+q+j)-2\alpha o-i-1}$$

$$H_{4,6}^{1,4}\left[\lambda_3^{-\beta}t^{\beta}\left|\begin{matrix}(1-(l+q+\zeta),0),((1-(-q+\zeta,0),(1-(-q-\zeta),1),(1-(j+\zeta+q),0)\\(0,1),(1-q-\zeta,0),(1+q,0),(1+\zeta+q,0),(1-\mathrm{q}-\zeta,0),\\(1-(\zeta+q+l-\beta(\zeta+q)+\alpha(\zeta+q+j)-2\alpha o-i,\beta)\end{matrix}\right.\right]$$

$$-\frac{2B}{d}\sum_{n=1}^{\infty}\sin(\frac{n\pi z}{d})(-(-1)^n+1)\sum_{j=0}^{\infty}\sum_{r=0}^{\infty}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{\zeta=0}^{\infty}\frac{(-1)^{k+\zeta+i+r+l+j}}{k!\,\zeta!\,l!\,j!}$$

$$y^k\left(\frac{1}{\varphi}\right)^{r+1}\delta^{2\zeta-1}\nu^{\zeta-q}N^l\sum_{o=0}^{j}\frac{j!}{o!\,(j-o)!}\lambda_3^{\beta(-q+\zeta)}\lambda_1^{\alpha(-j-\zeta+q)}\lambda_2^{\alpha o}t^{\zeta+q+l-\beta(\zeta-q)+\alpha(\zeta-q+j)-2\alpha o-1}$$

$$H_{4,5}^{1,4}\left[\lambda_3^{-\beta}t^{\beta}\left|\begin{matrix}(1-(1-q+\zeta),0),(1-(l+q+\zeta-1)\,,0),(1-(q-\zeta),1),(1-(j-q+\zeta)\,,0)\\(0,1),(1-(1-q)\,,0),(1-(q+\zeta-1),0),(1(-\zeta+q),0),(1-(\zeta-q)\,,0),\\(1-(q+\zeta+l-\beta(-q+\zeta)+\alpha(-q+\zeta+j)-2\alpha o,\beta)\end{matrix}\right.\right]$$

$$+\frac{2}{d}\sum_{n=1}^{\infty}\sin(\frac{n\pi z}{d})B(1-(-1)^n)\sum_{k=0}^{\infty}\sum_{\zeta=0}^{\infty}\sum_{L=0}^{\infty}\frac{(-1)^{k+\zeta+L}}{\zeta!\,L!}\delta^{-2(k+1)-1}\nu^{-k-1}N^{-\zeta}$$

$$\sum_{o=0}^{L}\frac{L!}{o!\,(L-o)!}\lambda_1^{\alpha(-k+L-1)}\lambda_2^{\alpha o}\lambda_3^{-\beta(k+1)}$$

$$t^{-\zeta-k-\beta(k-1)+\alpha(-k+L-1)-2\alpha o}$$

$$H_{3,5}^{1,3}\left[\lambda_3^{-\beta}t^{\beta}\left|\begin{matrix}(1-(\zeta-k),0),(1-(-k+L-1),0),(-k,1))\\(0,1),(1+k,0),(k+2,0),((\mathrm{k}+1),0),(1-(1-\zeta-k+\beta k+\beta+\alpha(-k+L-1)-2\alpha o,,\beta)\end{matrix}\right.\right] \quad \dots(21)$$

To obtain Equation (21), the following feature of the Fox H-function (18)is utilized:

$$\sum_{m=0}^{\infty}\frac{(-z)^m\prod_{i=0}^{k}\Gamma(a_i+A_im)}{m!\prod_{i=0}^{h}\Gamma(c_i+C_im)}=H_{k,s+1}^{1,k}\left[z\left|\begin{matrix}(1-a_1,A_1),\dots\dots,(1-a_k,A_k)\\(0,1),(1-c_1,C_1),\dots.,(1-c_s,C_s)\end{matrix}\right.\right]$$

Solution of Shear Stress

Utilized the Laplace transformation to Equation (10) and Equation (11), we get the equations

$$\tau 1=\frac{\mu\left(1+\lambda_3^{\beta}p^{\beta}\right)}{\left(1+\lambda_1^{\alpha}p^{\alpha}+\lambda_2^{\alpha}p^{2\alpha}\right)}\,\partial_y\mathrm{u}(y,z,p)\ \dots\ (22)$$

$$\tau 2=\frac{\mu\left(1+\lambda_3^{\beta}p^{\beta}\right)}{\left(1+\lambda_1^{\alpha}p^{\alpha}+\lambda_2^{\alpha}p^{2\alpha}\right)}\,\partial_z\mathrm{u}(y,z,p)\dots(23)$$

Applying the invers Finite Fourier sine transform to Equation (17)and source(17) we obtained $u(y,z,p)$ and substituting in to Equation(22),we get.

$$\tau 1=\frac{-(2)}{d}\mu\sum_{n=1}^{\infty}\sin\left(\frac{n\pi z}{d}\right)\left(\frac{H\dfrac{(1-(-1)^n)}{\delta(1+p)}}{1+\varphi\sqrt{\delta^2+\dfrac{p+N}{\nu H}}}\right).\left(\sqrt{\delta^2+\frac{N+p}{\nu H}}\right)Exp\left(-\sqrt{\delta^2+\frac{N+p}{\nu H}}y\right)$$
$$-\frac{\dfrac{HB(1-(-1)^n)}{\delta p\left(\nu\mathrm{H}\delta^2+(N+p)\right)}}{1+\varphi\sqrt{\delta^2+\dfrac{N+p}{\nu H}}}.\left(\sqrt{\delta^2+\frac{N+p}{\nu H}}\right)Exp\left(-\sqrt{\delta^2+\frac{N+p}{\nu H}}y\right) \quad \dots(24)$$

After performing calculations Equation (24) can be rewritten as follows:

$$\tau 1=\frac{-2}{d}\mu\sum_{n=1}^{\infty}sin\left(\frac{n\pi z}{d}\right)\Bigg((1-(-1)^n)\sum_{\gamma=0}^{\infty}\sum_{r=0}^{\infty}\sum_{\sigma=0}^{\infty}\sum_{l=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{j=0}^{\infty}\sum_{a=0}^{\infty}\frac{(-1)^{r+\gamma+\sigma+i+\omega+j+l}}{\gamma!\,\sigma!\,l!\,\omega!\,j!}y^{\gamma}\left(\frac{1}{\varphi}\right)^{r+1}$$
$$\delta^{2\sigma-1}\nu^{\sigma+\eta+\frac{1}{2}}N^l\frac{\Gamma(-\eta+\sigma-1/2)}{\Gamma(-(\eta+1/2))}\frac{\Gamma(\eta+\sigma+l+1/2)}{\Gamma(\eta+\sigma+1/2)}\frac{\Gamma(\omega-\eta-\sigma-3/2)}{\Gamma(-\eta-\sigma-3/2)}\frac{\Gamma(j+\eta+\sigma+3/2)}{\Gamma(\eta+\sigma+3/2)}$$
$$\sum_{i=0}^{j}\frac{j!\ \lambda_1^{-\alpha(\eta+\sigma+3/2+j)}}{i!\,(j-i)!}\lambda_2^{\alpha i}\lambda_3^{\beta\left(\sigma+\eta+\frac{3}{2}-\omega\right)}p^{a+\alpha(2i-\sigma-\eta-\frac{3}{2}-j)+\beta\left(\sigma+\eta+\frac{3}{2}-\omega\right)-(\sigma+\eta+\frac{1}{2}+l)}$$
$$-B(1-(-1)^n)\sum_{\gamma=0}^{\infty}\sum_{r=0}^{\infty}\sum_{\sigma=0}^{\infty}\sum_{l=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^{r+\gamma+\sigma+l+\omega+j}}{j!\,\gamma!\,\sigma!\,l!\,\omega!}y^{\gamma}\left(\frac{1}{\varphi}\right)^{r+1}\delta^{2\sigma-1}\,\nu^{\sigma-\eta-1/2}N^l$$
$$\frac{\Gamma(\sigma+1/2-\eta)}{\Gamma(1/2-\eta)}\frac{\Gamma(\omega+\eta-\sigma-1/2)}{\Gamma(\eta-\sigma-1/2)}\frac{\Gamma(j+\sigma+1/2-\eta)}{\Gamma(\sigma+1/2-\eta)}\frac{\Gamma(l+\eta+\sigma-1/2)}{\Gamma(\eta+\sigma-1/2)}\sum_{a=0}^{j}\frac{j!\,\lambda_1^{-\alpha(j+\sigma+1/2-\eta)}}{a!\,(j-a)!}$$
$$(\lambda_2^{\alpha a})\ \lambda_3^{\beta(\sigma+1/2-\eta-\omega)}p^{-1-\alpha\left(j+\sigma+\frac{1}{2}-\eta+2\alpha a\right)+\beta\left(\sigma+\frac{1}{2}-\eta-\omega\right)-l-\sigma+1/2-\eta}\Bigg) \quad \dots(25)$$

Applying the inverse Laplace transformation to Equation (25), we obtain the solution of shear stress

$$\tau 1=\frac{-2}{d}\mu\sum_{n=1}^{\infty}sin\left(\frac{n\pi z}{d}\right)\Bigg((1-(-1)^n)\sum_{\gamma=0}^{\infty}\sum_{r=0}^{\infty}\sum_{\sigma=0}^{\infty}\sum_{l=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{j=0}^{\infty}\sum_{a=0}^{\infty}\frac{(-1)^{r+\gamma+\sigma+l+\omega+j+i}}{\gamma!\,\sigma!\,l!\,\omega!\,j!}\ y^{\gamma}\left(\frac{1}{\varphi}\right)^{r+1}\delta^{2\sigma-1}$$
$$\nu^{\sigma+\eta+\frac{1}{2}}N^l\ \frac{\Gamma(\sigma-1/2-\eta)}{\Gamma(-1/2-\eta)}\frac{\Gamma(l+\eta+\sigma+1/2)}{\Gamma(\eta+\sigma+1/2)}\frac{\Gamma(\omega-\sigma-3/2-\eta)}{\Gamma(-\sigma-3/2-\eta)}\frac{\Gamma(j+3/2+\sigma+\eta)}{\Gamma(3/2+\sigma+\eta)}\sum_{i=0}^{j}\frac{j!}{i!\,(j-i)!}$$

$$\lambda_1^{-\alpha(3/2+\sigma+\eta+j)}\lambda_2^{\alpha i}\lambda_3^{\beta(3/2-\omega+\sigma+\eta)}\frac{t^{(-1/2+\sigma+\eta+l)-a-\alpha(2i-3/2-j-\sigma-\eta)-\beta(3/2+\sigma+\eta-\omega)}}{\Gamma\big(1/2+\sigma+\eta+l-a-\alpha(2i-3/2-j-\sigma-\eta)-\beta(3/2+\sigma+\eta-\omega)\big)}$$
$$-B(1-(-1)^n)\sum_{\gamma=0}^{\infty}\sum_{r=0}^{\infty}\sum_{\sigma=0}^{\infty}\sum_{l=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^{r+\gamma+\sigma+l+\omega+j}}{\gamma!\,\sigma!\,l!\,\omega!\,j!}y^{\gamma}\left(\frac{1}{\varphi}\right)^{r+1}\delta^{2\sigma-1}\,\nu^{\sigma-\eta-\frac{1}{2}}\,\frac{\Gamma\left(\sigma-\eta+\frac{1}{2}\right)}{\Gamma\left(-\eta+\frac{1}{2}\right)}$$
$$\frac{\Gamma(\omega-\sigma+\eta-1/2)}{\Gamma(-\sigma+\eta-1/2)}\frac{\Gamma(j+\sigma-\eta+1/2)}{\Gamma(\sigma-\eta+1/2)}\frac{\Gamma(l+\sigma+\eta-1/2)}{\Gamma(\sigma+\eta-1/2)}N^l\sum_{a=0}^{j}\frac{j!}{a!\,(j-a)!}\lambda_1^{-\alpha(j+\sigma-\eta+1/2)}\,(\lambda_2^{\alpha a})$$
$$\lambda_3^{\beta(\sigma-\eta+1/2-\omega)}\left.\frac{t^{\alpha(j+1/2+2\alpha a+\sigma-\eta)-\beta(1/2-\omega+\sigma-\eta)+l-1/2+\sigma+\eta}}{\Gamma(\alpha(j+1/2+2\alpha a+\sigma-\eta)-\beta(1/2-\omega+\sigma-\eta)+l-1/2+\sigma+\eta)}\right)\ldots(26)$$

We can obtain the form of Equation (26) by using the Fox H-function(18).

$$\tau 1=\frac{-2}{d}\mu\sum_{n=1}^{\infty}\sin\left(\frac{n\pi z}{d}\right)\Big((1-(-1)^n)\sum_{\gamma=0}^{\infty}\sum_{r=0}^{\infty}\sum_{\sigma=0}^{\infty}\sum_{a=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^{r+\gamma+\sigma+l+j+i}}{l!\,\gamma!\,\sigma!\,j!}\,y^{\gamma}\left(\frac{1}{\varphi}\right)^{r+1}$$
$$\delta^{2\sigma-1}\,\nu^{\sigma+\eta+\frac{1}{2}}\,(N^l)\sum_{i=0}^{j}\frac{j!}{i!\,(j-i)!}\,\lambda_1^{-\alpha\left(\frac{3}{2}+j+\sigma+\eta\right)}\lambda_2^{\alpha i}\,\lambda_3^{\beta\left(\frac{3}{2}+\sigma+\eta\right)}t^{\left(-\frac{1}{2}+\sigma+\eta+l\right)-a-\alpha\left(2i-\frac{3}{2}-j-\sigma-\eta\right)-\beta\left(\frac{3}{2}+\sigma+\eta-\omega\right)}$$
$$H_{4,6}^{1,4}\left[\lambda_3^{-\beta}t^{\beta}\left|\begin{matrix}(1-(-1/2+\sigma-\eta),0),(1-(\sigma+\eta+l+\frac{1}{2}),0),(1-(-\frac{3}{2}-\sigma-\eta),1),(1-(j+\frac{3}{2}+\sigma+\eta),0)\\(0,1),(1-(-\frac{1}{2}-\eta),0),(1-(\frac{1}{2}+\sigma+\eta),0),(1-(-\frac{3}{2}-\sigma-\eta),0),(1-(\frac{3}{2}+\sigma+\eta),0),\\(1-((\frac{1}{2}+l+\sigma+\eta)-a-\alpha(2i-\frac{3}{2}--\sigma-\eta-j)-\beta(\frac{3}{2}+\sigma+\eta),\beta)\end{matrix}\right.\right]$$
$$-B(-(-1)^n+1)\sum_{l=0}^{\infty}\sum_{r=0}^{\infty}\sum_{\sigma=0}^{\infty}\sum_{\gamma=0}^{\infty}\sum_{\omega=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^{r+\gamma+\sigma+l+\omega+j}}{\gamma!\,\sigma!\,l!\,\omega!\,j!}y^{\gamma}\left(\frac{1}{\varphi}\right)^{r+1}\delta^{2\sigma-1}\,\nu^{\sigma-\eta-\frac{1}{2}}\,N^l$$
$$\sum_{a=0}^{j}\frac{j!}{a!\,(j-a)!}\lambda_1^{-\alpha\left(j+\frac{1}{2}+\sigma-\eta\right)}\,(\lambda_2^{\alpha a})\,\lambda_3^{\beta\left(\frac{1}{2}+\sigma-\eta\right)}t^{\alpha\left(j+\frac{1}{2}+2\alpha a+\sigma-\eta\right)-\beta\left(\frac{1}{2}+\sigma-\eta\right)+l-\frac{1}{2}+\sigma+\eta}$$
$$H_{4,6}^{1,4}\left[\lambda_3^{-\beta}t^{\beta}\left|\begin{matrix}(1-(\frac{1}{2}+\sigma-\eta),0),(1-(-\frac{1}{2}-\sigma+\eta),1),(1-(j+\frac{1}{2}+\sigma-\eta),0),(1-(l-\frac{1}{2}+\sigma+\eta),0)\\(0,1),(1-(\frac{1}{2}-\eta),0),(1+\frac{1}{2}-(-\sigma+\eta),0),(1+\frac{1}{2}-(\sigma-\eta),0),(1+\frac{1}{2}-(\sigma+\eta),0),\\(1-(\alpha(j+\frac{1}{2}+\sigma-\eta+2\alpha a)-\beta(\sigma+\frac{1}{2}-\eta)+l+\frac{1}{2}+\sigma+\eta),\beta)\end{matrix}\right.\right]$$

As to the shear stress $\tau 2$, it can be obtained from Equation (23) and Equation (17)by implementing the same count steps with those of $\tau 1$

Analysis and Results

That work, we discussed the analytic solution for the magnatohydrodynamic flow of Burgers' fluid with fractional derivative because of a constant pressure gradient and exponential acceleration plate between two sides wall. That be by means of the Finite Fourier sine and Laplace transformations, so the solutions are acquired in terms of the Fox H-function. Several Figures are drawn to display the influence of various parameters of the Burger's fluid , Fig.1 shows the effects of increasing magnetic field N results in the increasing of the velocity surfaces .

Figure 2, 3 the variation of the fractional derivative of parameters α,β that show ,the effects of increasing α is retard the velocity increasing surface, so increasing β has the adverse effect of α. Figure 4, 5 show the material parameters λ_1, λ_3, the effect of decreasing λ_1 (λ_3 increase) is the increasing velocity surface that generalized burger's fluid. Figure 6 show the difference of velocity surface of different value of time.

It is apparent that the velocity flow is increase with increase of time *t*. Figure 7 displays influence of slip coefficient ϕ, the fluid flows increase with decreasing slip coefficient ϕ.

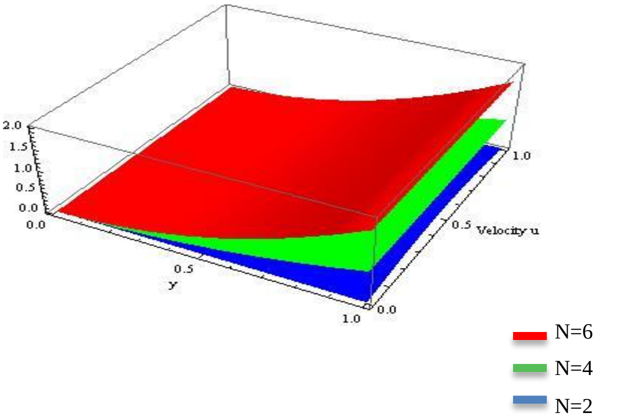
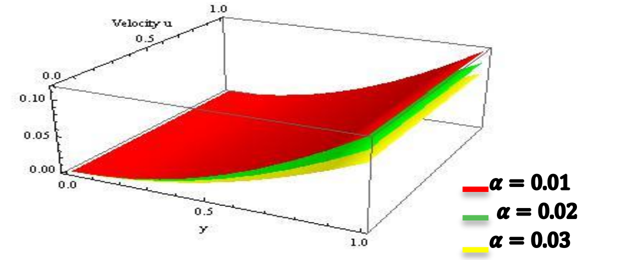
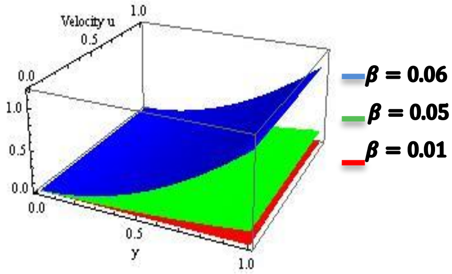


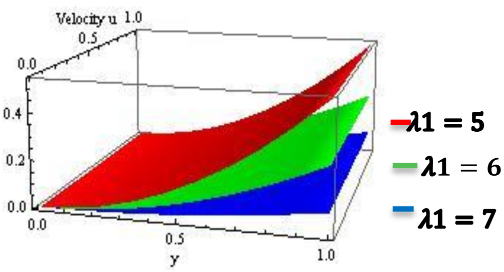
Figure 1. Velocity u for (N=6,4,2)when keeping,ϕ, λ_1 ,t, λ_2 , λ_3 ,α,β,fixed.



Figurer 2. velocity surface for(α =0.01, 0.02, 0.03) when keeping ϕ, λ_1 ,t, λ_2 , λ_3 , N,β,fixed



Figurer 3. velocity surface for (β =0.06 ,0.05 ,0.01)when keeping,α, ϕ, λ_1 ,t, λ_2 , λ_3 , N, fixed



Figurer 4. velocity surface for (λ_1 =5,6,7)when keeping,α, ϕ, β ,t, λ_2 , λ_3 , N, fixed

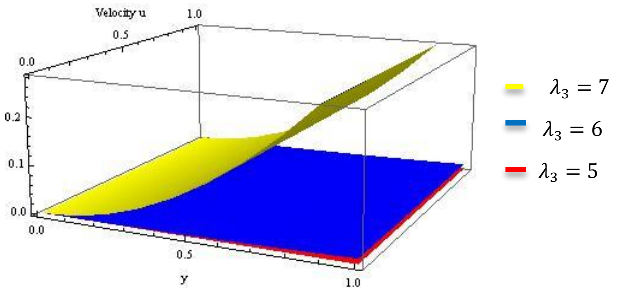


Figure 5. velocity surface for ($\lambda_3 = 7, 6, 5$) when keeping, α, ϕ, λ_1, β ,t, λ_2 , N, fixed

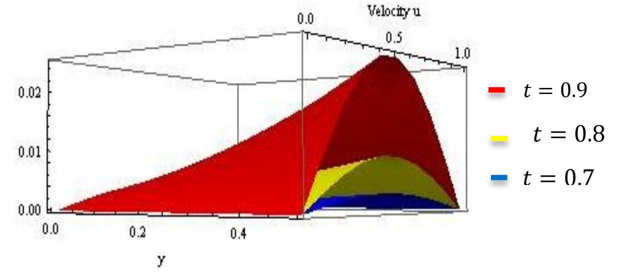


Figure 6. velocity surface for (t =0.9,0.8,0.7) when keeping, α, ϕ, λ_1, β ,λ_2 , λ_3 , N, fixed

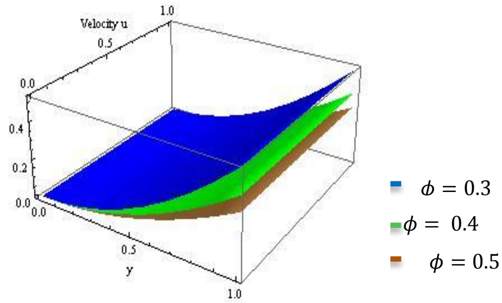


Figure 7. velocity surface for($\phi = 0.3, 0.4, 0.5$) when keeping,α, λ_1, β ,λ_2 , λ_3 , N, t, fixed

Conflicts of Interest: None.

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تأثير الحقل المغناطيسي الهيدروديناميكي ومعامل الانزلاق لمائع بيركر ذو المشتقات الكسرية

آمال محي نصيف[1] **محمد علي مراد[2]**

[1]قسم الرياضيات، كلية العلوم، جامعة ديالى، ديالى، العراق
[2] قسم الرياضيات، كلية التربية الاساسية ، جامعة ديالى، ديالى، العراق.

الخلاصة:
هذا البحث يهدف الى تاثير حقل مغناطيسي هيدروديناميكي للمائع بيركر القابل للانضغاط من خلال ضغط ثابت ولوح متسارع أسي. حيث افتراض عدم الانزلاق بين لوحة التسارع والمائع . حساب التفاضل والتكامل الكسري استخدم لكتابة معادلات الحركة لنموذج المائع ، باستخدام تحويلات لابلاس وفوريير نحصل على حقل السرعة والاجهاد. اضافة الى ذلك، تم رسم الاشكال الثلاثية الابعاد لعرض تاثير حقل المغناطيسي والمعلمات المختلفة على حقل السرعة.

الكلمات المفتاحية: مائع بيركر، دالة Fox- H ، تحويل لابلاس ، معامل الانزلاق.