Forecasting the Dollar/LD Exchange Rate in Libyan Parallel Market

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Abstract
Forecasting is an important tool for decision making. It is utilized, In particular, by stakeholders such as dealers and risk managers to decide about the potential exchange rate transactions. Our aim is to forecast the behavior of Dollar/LD foreign exchange rate in a predetermined target horizon using Monte Carlo simulation. Modeling procedures are focused mainly on Brownian motion. Inferences about mean exchange rate and value-at-risk at the target horizon are also considered.

Keyword. Forecasting – Foreign exchange rate – Parallel market - Monte Carlo simulation— Brownian motion - Value-at-Risk.

الملخص
التنبؤ هو أداة مهمة في اتخاذ القرارات، يوظفه المستفيدون كالتجار ومديرو المخاطر في اتخاذ القرارات حيال صفقات بيع العملة. نهدف في هذه الورقة الى التنبؤ بسلوك سعر صرف الدولار بالدينار الليبي في السوق الليبي الموازي خلال أفق زمني مستهدف **target horizon**، مستخدمين في ذلك طريقة مونتي كارلو monte Carlo للمحاكاة. سوف نقوم بالتركيز في نمذجة المحاكاة بالتحديد على حركة براون Brownian motion.كما سنقوم أيضاً بإجراء استنتاجات حول متوسط سعر الصرف في الافق الزمني وكذلك حول القيمة العرضة للخطر value at risk.

1 Introduction
Exchange rate is the number of units of domestic currency which can be exchanged for one unit of foreign currency (Cipra, 2010). In order to make decisions about the potential exchange rate transactions, stakeholders should be aware about the behavior of the underlying market movements. The objective is to forecast the random behavior of Dollar/LD exchange rate during a target horizon of the next thirty days. This has been achieved through Monte Carlo simulations of geometric Brownian motion. Value-at-Risk (VaR) and mean exchange rate are also inferred during the prescribed target horizon.
The data used is a historical time series of Dollar/LD exchange rate during the time period 1.1.2020 - 8.3.2020[1] obtained from AL-Mushir Foreign Exchange, FOREX, parallel market of Tripoli[2]. Ad hoc procedures[3] have been employed to deal with the weekend effect and missing

[1] see Appendix 1.

[2] Source: ewanlibya.ly.

[3] For more information refer to Reiss & Thomas, 2007. pp 373.

exchange rates. 68 return rates are computed from the historical data from which the mean return rate and volatility are estimated to be used further in forecasting.
In the sequel, starting with the Gaussian distribution, we introduce furthermore the Brownian motion in an increasing order of complexity.

2 Gaussian Modeling

2.1 Gaussian Distribution

A continuous random variable X is said to be Gaussian distributed, symbolically $X\sim N(\mu,\sigma^2)$, if and only if its density function is defined as

$$\varphi_{\mu,\sigma}(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\},-\infty<x<\infty$$

with distribution function

$$\Phi_{\mu,\sigma}(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(t-\mu)^2}{2\sigma^2}\right\}dt$$

A special case of special importance is the standard Gaussian for which $\mu=0,\sigma^2=1$, that is $\Phi_{0,1}(z)$.

2.2. Gaussian Data Generation

A set of n standard Gaussian data values $z_i, i=1,2,\dots,n$ can be generated in such a way that satisfies;

$$\{z_i\colon \Phi_{0,1}(z_i)\geq q\}_{i=1}^{n}, 0<q_i<1$$

This is attained if $z_i=\Phi_{0,1}^{-1}(q_i), i=1,2,\dots,n$ where $\Phi_{0,1}^{-1}$ is the direct inverse of $\Phi_{0,1}$. Interesting properties of Gaussian distribution are the symmetry and mesokurtosis. Figure 1. shows the characteristics of standard Gaussian distribution with generated standard Gaussian random variable.

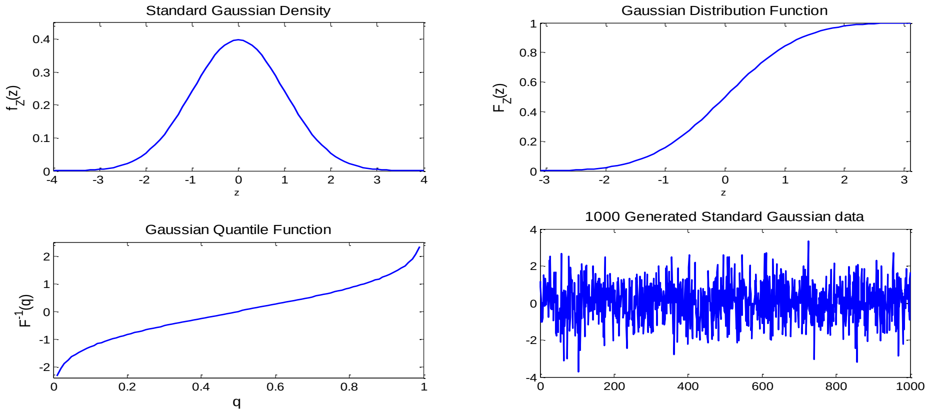

Figure 1. standard Gaussian Distribution.

3 Stochastic Variables

Any variable whose value changes over time , either discrete or continuous, in an uncertain way is said to follow a stochastic process. In financial markets, exchange rate is considered a typical random variable. Following, we introduce Markov process as a basis for modeling the future behavior of the Dollar/LD exchange rate. In order to avoid violating the modeling assumption, awareness, such as the shape of the distribution, parameterization, etc., has been taken into account in implementation.

3.1 Markov Process

Markov process is a kind of stochastic processes so that its predictions for the future should be unaffected by the past. That is it is independent of its history and relies only on the current value.

3.2 Wiener Process

A stochastic process $\{X_t : t \geq 0\}$ is called a Wiener process if it satisfies the following properties;

1. $X_0 = 0$, i.e., starts from state zero.
2. the increments $\Delta X = X_{t+1} - X_t$ are independent.
3. The increments, ΔX are stationary so that

$$\Delta X = \mu\,\Delta t + \sigma\sqrt{\Delta t}\,z$$

Is Gaussian distributed with drift $\mu\Delta t$ and variance$\sigma^2\Delta t$ proportional to the time interval, Δt. To simulate ΔX we need the quantile function $\Phi^{-1}_{\mu\Delta t,\sigma\sqrt{\Delta t}}(q)$,for different values of probability q. In terms of standard Gaussian quantile function $\Phi^{-1}_{0,1}$ one can implement the relation

$$\Phi^{-1}_{\mu\Delta t,\sigma\sqrt{\Delta t}}(q) = \mu\,\Delta t + \sigma\sqrt{\Delta t}\,\Phi^{-1}_{0,1}(q)$$

The property of statistical independence and identical distribution of increments form the corner stone that leads to the well-known Wiener process (Tapiero, 2004), and its generalization, *Ito* process, for which geometric Brownian motion $\{S_t : t \geq 0\}$ is a particular case, namely

$$\Delta S = S\,\Delta X = S\left(\mu\,\Delta t + \sigma\sqrt{\Delta t}\,z\right)$$

Obviously, $\frac{\Delta S}{S} \sim N(\mu\,\Delta t, \sigma^2 \Delta t)$. The process is geometric of type in that each of the drift $\mu \Delta t$ and volatility $\sigma\sqrt{\Delta t}$ are proportional to the current value S_0. The result also explains why uncertainty, as measured by volatility, is sometimes referred to as being proportional to the square root of the time interval ahead (Hull, 2012. This ensures positive values of the random variable S.

4. Implementing Simulations

Simulation is the process of creating synthetic data with desired properties. it is a central technique for risk managers to create samples with characteristics similar to those of underlying risk factors. Monte Carlo simulation[4] has become an increasingly popular and necessary technique for business analysis and forecasting. The techniques are, nowadays, becoming more flexible and easier to implement with the advancing aids of computing and software packages.

4.1 Dollar/LD Exchange Rate Modeling

The first step of modeling starts with computing the rate of return r_t of the underlying historical exchange rate $P_t, t = 1, \ldots, 68$. By using the formula

$$r_{t+1} = \frac{P_{t+1} - P_t}{P_t}, t = 1,2, \ldots, 68$$

for which the estimated mean rate and volatility are found to be $\hat{\mu}_r = 00.16\%$ and $\hat{\sigma}_r = 1.65\%$ respectively.

As above mentioned, the speculative exchange rates P_t are governed by a discrete time geometric Brownian motion $\{P_t : t = 0,1,2, \ldots\}$. If the current exchange rate is P_0, then we are encountered with the stochastic model

$$P_{t+1} = P_t + P_0\left(0.0016\,\Delta t + 0.0165\sqrt{\Delta t}\,Z\right), i = 1,2,3, \ldots, 30$$

with time interval $\Delta t = 30\left(\frac{1}{65}\right)$ and Z is a standard Gaussian random variable. Simulation output of potential exchange rate of the next thirty days, for a current exchange rate of $P_0 = 4.54$ as of 8.3.2020, is as given on Table 1.

[4] Monte Carlo simulation was named for Monte Carlo, Monaco, where the primary games of chance were held (Goldman,2000).

Table 1. Monte Carlo Simulation of future Dollar/LD Exchange Rate

	Random Variables		Dollar/LD Exchange Rate	
Day i	q_i	ΔX	Increment ΔP	Future Price P_{i+1}
0				4.5400
1	0.2564	-0.0065	-0.0295	4.5105
2	0.6135	0.0039	0.0177	4.5282
3	0.5822	0.0030	0.0137	4.5418
⋮	⋮	⋮	⋮	⋮
28	0.6279	0.0043	0.0196	4.6433
29	0.7720	0.0089	0.0406	4.6840
30	0.9329	0.0172	0.0783	4.7623

It is of interest that as the exchange rate falls the variance decreases which makes it unlikely to experience a downmove, that would push the exchange rate to negative values, the principal assumption of the underlying process modeling.

4.2 **Inferences about the Target Horizon Parameters**

Monte Carlo Inference about the target horizon passes through three consecutive steps, namely, risk factors specification, process determination and making inferences about the target horizon data (Crouhy et al, 2001).

4.2.1**. Mean Exchange Rate**

In order to estimate the target horizon mean exchange rate, the process has to be replicated as many times as needed. If the process is replicated m times, then there should be m target horizon exchange rate values, namely, $P_{T,k}, k = 1,2, \dots, m$. Figure 2. Shows four replicated scenarios of the exchange rate process.

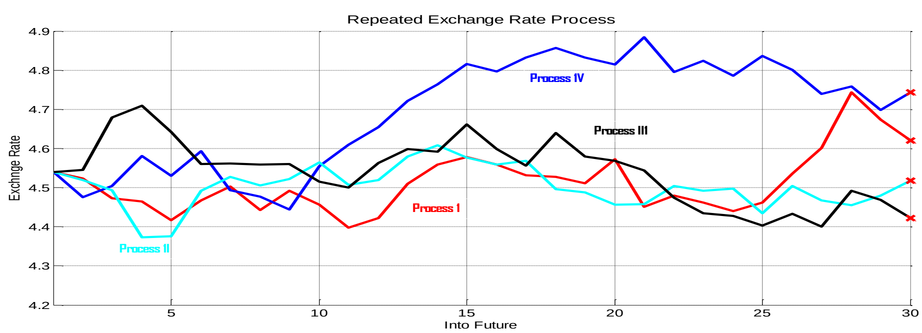

Figure 2. Replicated Geometric Brownian motion of Dollar/LD exchange price

Our objective is to estimate the mean target price $\mu_{P_T} = M^{-1}\sum_{j\leq N} P_{T,j}$ of a population of size M using a confidence interval on the basis of a sample mean $\bar{P}_T = m^{-1}\sum_{k\leq N} P_{T,k}$ of a sample of size m. If the sample is large enough $(m \geq 32)$, the central limit theorem would play an important role so that, in the limiting, standardized target horizon sample mean

$$\frac{\bar{p}_T - \mu_{\bar{p}_T}}{S_{\bar{p}_T}/\sqrt{m}}$$

is a standard Gaussian (Freund, 1992), Table 2. shows the maximum likelihood estimates for the respective parameters and Table 3. Shows the estimated covariance of the estimates.

Table 2. Gaussian Fitting to the Target Horizon Exchange Rate P_{30}

Parameter	Estimate	Std. Err
μ	4.63535	0.00846649
σ	0.267734	0.00599121

Table 3. Estimated covariance of parameter estimates

	$\hat{\mu}_{P_{30}}$	$\hat{\sigma}_{P_{30}}$
$\hat{\mu}_{P_{30}}$	7.16815e-05	2.54381e-18
$\hat{\sigma}_{P_{30}}$	2.54381e-18	3.58946e-05

Applying the $(1-\alpha)100\%$ confidence interval

$$\bar{P}_T \mp Z_{\frac{\alpha}{2}}\frac{S_{\bar{p}_T}}{\sqrt{n}}$$

With α as level of significance and a tabulated value $Z_{\frac{\alpha}{2}}$ such that

$$Pr\left(Z < Z_{\frac{\alpha}{2}}\right) = 1 - \frac{\alpha}{2}.$$

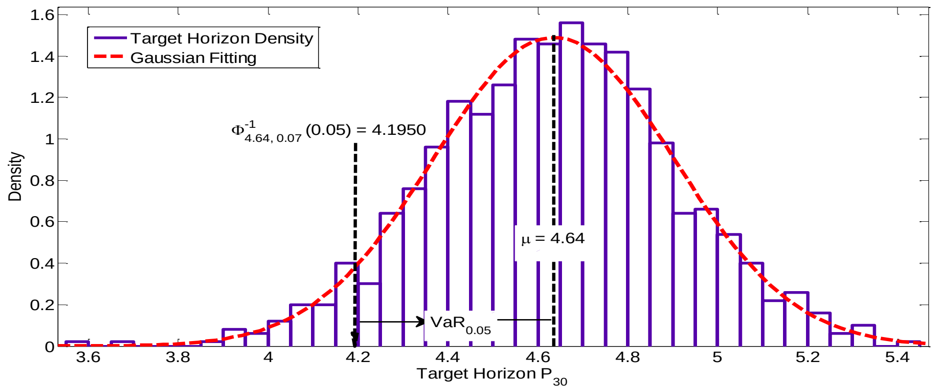

Figure 3. Gaussian modeling (red) of the exchange rate at target horizon (blue) with related Value-at-Risk.

One thousand replicates of the exchange rate process produces a 95 % confidence interval of the target horizon ($T = 30$), namely, $4.6226 \leq \bar{P}_{30} \leq 4.6559$.

4.2.2. **Value-at-Risk**

Value-at-Risk (VaR) is a summary statistic that quantifies the exposure to market risk, and hence used as a measure of risk (Warwick, 2003). Mathematically, VaR_q is the deviation of the q-quantile from the mean of the underlying distribution curve (Jorion, 2003). That is, for a normal random variable X , the value at risk is defined as

$$VaR_q = E(X) - \Phi^{-1}_{\mu,\sigma}(q)$$

It can be interpreted as the threshold such that a loss will not happen with a specified probability. In other words, there is a confidence of $(1-q)100\%$ that the loss over the target horizon will not exceed VaR_q.

An advantage of Monte Carlo value at risk is that it can capture a wider range of market behavior as well as capturing risk arising from scenarios that do not involve extreme market moves (Alexander & Sheedy, 2011).

Considering a target horizon exchange rate of thirty days, namely, P_{30} , we are looking for the estimate

$$\widehat{VaR}_q = \bar{P}_{30} - \Phi^{-1}_{\hat{\mu},\hat{\sigma}}(q)$$

for different values of q. Due to the effect of sampling variability on the precision of estimates, the number of replications should be large enough. According to (Jorion,2003), using 1000 replicates will result in a 95% estimated value at risk, $\widehat{VaR}_{0.05}$ that is more precise in contrast with $\widehat{VaR}_{0.01}$.

For the purpose, Table 4. shows the estimated Value-at-Risk of underlying target horizon distribution for $q = 0.05, 0.01, 0.005, 0.001$. It shows a confidence of 95% that, on the target horizon, the loss per unit dollar will not exceed 0.45 LD ,approximately, other values can be interpreted in similar fashion.
It is of interest that Reiss, R-D and Thomas, M (Reiss & Thomas, 2007) and McNeil, A. (McNeil, 1999), looked at the value at risk VaR_q as the q-quantile of the underlying curve rather than a deviation from the mean value.

Table 4 . Simulated Value at Risk (VaR_q) of 1000 replicates of exchange rate at target time T=30.

Value at Risk (VaR_q)				
q	0.95	0.99	0.995	0.999
VaR_q	0.4426	0.6260	0.6931	0.8315

5. Discussion and Conclusion

As mentioned above, modeling procedure is, generally, not robust to assumption violation and, furthermore, depends on the horizon considered and hence can be misleading. Fortunately, for short target horizon, the choice of the Gaussian assumption does not really matter against the choice of other literature modeling such as the logGaussian. In this work, as the process is approximated by small steps, namely days, this supports the reliability of our choice of Gaussian modeling.
The random behavior of exchange rate shows a tendency to rise during the next thirty days. Inferences show a 95% confidence that the mean exchange rate will not be less than 4.6 LD., in the target horizon. Inferences about value at risk also show that a maximum loss exceeding 0.83 dinar per unit dollar may occur but with a small probability that does not exceed 0.001.

6. References

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7. Appendices

Appendix1.Dollar/LD exchange rates, Al-Mushir Market

Day	Date	Rate
Wed	1-1-2020	4.12
Thu	2-1-2020	4.11
Fri	3-1-2020	4.13
Sat	4-1-2020	4.15
Sun	5-1-2020	4.15
Mon	6-1-2020	4.15
Tue	7-1-2020	4.14
Wed	8-1-2020	4.14
Thu	9-1-2020	4.14
Fri	10-1-2020	4.13
Sat	11-1-2020	4.12
Sun	12-1-2020	4.12
Mon	13-1-2020	4.50
Tue	14-1-2020	4.20
Wed	15-1-2020	4.20
Thu	16-1-2020	4.20
Fri	17-1-2020	4.20
Sat	18-1-2020	4.20
Sun	19-1-2020	4.22
Mon	20-1-2020	4.32
Tue	21-1-2020	4.32
Wed	22-1-2020	4.30
Thu	23-1-2020	4.30
Fri	24-1-2020	4.32
Sat	25-1-2020	4.35
Sun	26-1-2020	4.35
Mon	27-1-2020	4.38
Tue	28-1-2020	4.39
Wed	29-1-2020	4.39
Thu	30-1-2020	4.39
Fri	31-1-2020	4.38
Sat	1-2-2-2020	4.37
Sun	2-2-2020	4.37
Mon	3-2-2020	4.37
Tue	4-2-2020	4.36
Wed	5-2-2020	4.33
Thu	6-2-2020	4.32
Fri	7-2-2020	4.32
Sat	8-2-2020	4.32

Sun	9-2-2020	4.30
Mon	10-2-2020	4.29
Tue	11-2-2020	4.29
Wed	12-2-2020	4.29
Thu	13-2-2020	4.29
Fri	14-2-2020	4.28
Sat	15-2-2020	4.27
Sun	16-2-2020	4.26
Mon	17-2-2020	4.26
Tue	18-2-2020	4.26
Wed	19-2-2020	4.26
Thu	20-2-2020	4.26
Fri	21-2-2020	4.26
Sat	22-2-2020	4.26
Sun	23-2-2020	4.26
Mon	24-2-2020	4.27
Tue	25/2/2020	4.27
Wed	26-2-2020	4.25
Thu	27-2-2020	4.27
Fri	28-2-2020	4.27
Sat	29-2-2020	4.27
Sun	1-3-2020	4.27
Mon	2-3-2020	4.50
Tue	3-3-2020	4.50
Wed	4-3-2020	4.50
Thu	5-3-2020	4.50
Fri	6-3-2020*	4.50
Sat	7-3-2020	4.50
Sun	8-3-2020	4.54

*. Rates of underline dates are estimated.

Appendix 2. Matlab Code of Simulation Processing

```
clc
figure
% n: # of days of time interval dt.
%m: # repeated processes
for l=1:m
q=rand(n,1);
dZ=norminv(q,0,1);
dZ1=norminv(q,mu*dt,sigma*sqrt(dt));
dX=mu*dt+sigma*sqrt(dt)*dZ;
dP=P0*dX;
P1=P0;
for k=1:n-1
   P1(k+1)=P1(k)+dP(k+1);
end
plot(P1,'k'), hold on
```

```
plot(n,P1(end),'xb','MarkerSize',12)
xlabel('Into Future')
ylabel('Exchnge Rate')
title('Repeated Exchange Rate Process')
d(l)=P1(end);
end
hold off
fprintf(' i     q~U(0,1)    dX      dP      P(t+1)\n');
disp('=============================')

fprintf('%3d     %1.4f     %1.4f     %1.4f     %1.4f
\n',0,0,0,0,P0);
for k=1:n
   fprintf('%3d    %1.4f    %1.4f    %1.4f    %1.4f\n',k, q(k),
dX(k), dP(k), P1(k));
end
zd=(d-mean(d))/std(d);
figure,
hist(zd);
% fit zd using "dfitool"
xlabel('Exchange Rate')
ylabel('Freq')
title('Histogram of EX Price on Target Horison')
[MUHAT,SIGMAHAT,MUCI,SIGMACI] = normfit(d);
disp('=============================')
disp('a 95% CI for MU is')
MUCI
disp('=============================')
VaRq=[0.99 0.995 0.999 0.9995;...
(mean(d)-norminv(0.99,mean(d),std(d))) (mean(d)-
norminv(0.995,mean(d),std(d))) (mean(d)-
norminv(0.999,mean(d),std(d)))...
   (mean(d)-norminv(0.9995,mean(d),std(d)))];
disp('Value at Risk for different values of q is as follows')
VaRq
disp('implemetation results may differ from the those of the
paper due to stochastic modeling')
```