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EMPIRICAL DISTRIBUTION OF STOCK RETURNS OF SOUTHEAST EUROPEAN EMERGING MARKETS

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Abstract

The assumption that equity returns follow the normal distribution, most commonly made in financial economics theory and applications, is strongly rejected by empirical evidence presented in this paper. As it was found in many other studies, we confirm that stock returns follow a leptokurtic distribution and skewness, which in most of the Southeast European (SEE) markets is negative. This paper investigates further whether there is any distribution that may be considered an optimal fit for stock returns in the SEE region. Using daily, weekly and monthly data samples for a period of five years from ten Southeast European emerging countries, we applied the Anderson-Darling test of Goodness-of-fit. We strongly rejected the aforementioned assumption of normality for all considered data samples and found that the daily stock returns are best fitted by the Johnson SU distribution whereas for the weekly and monthly stock returns there was not one predominant, but many distributions that can be considered a best fit.

Keywords: Goodness-of-fit test, Anderson-Darling test, Johnson SU distribution.

Jel Classification: C10; C11; G10

INTRODUCTION

The type of distribution that is implicitly or explicitly incorporated in the models in the theory and application in the financial economics is crucial. The most famous models which are now used as are the portfolio theory of Markowitz, CAPM, Black-Sholes etc., are based on the assumption for normal distribution. But empirical evidence has long shown that the assumption of the normal distribution is not a consistent explanation for the stock returns. The normal distribution implies that the preferences of investors are

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modelled in a simple way by assuming mean-variance behavior, together with the implausibility of quadratic utility functions. This means that the flow of information on the stock markets is linear and that the reactions of investors to such information is also linear. Peters (1991) demonstrates that the information on the stock markets come with infrequent clumps instead of in a linear fashion, and investors at least infrequently respond to them, but they can also not react. Therefore, if the flow of information is leptokurtic, then stock returns will have leptokurtic distribution.

Another requirement to fulfill the assumption of the normal distribution is that equity markets are rational and efficient. According to that logic, if return expectations implicit in asset prices are rational, actual rates of return should be normally distributed around these expectations (Bodie et al. 2014, 135). But empirical evidence strongly rejects normal distribution and shows that stock returns have leptokurtic distribution and skewness (somewhere on the left, somewhere on the right).

In this paper we want to determine the best fitted distribution of the daily, weekly and monthly stock returns for 10 emerging markets of Southeast Europe. Most of the studies of this type are based on research on the case of developed markets like those of Western Europe and the USA. Research of the emerging markets, especially in the case of SEE markets, are very rare. The time series used in our analysis starts from July 1, 2011 to June 30, 2016, and we intentionally skip the period of the great financial crisis of 2007/08 and the European debt crisis of 2009/10. Actually, in our first analysis we included that period as well, but no distribution can be fitted in all cases. That is because the fluctuations in those periods was extremely huge, and afterwards a pretty calm period followed.

We have no ex-ante underlying financial theory to justify the use of all the specifications. Moreover, our purpose is to fit distributions that allow for the characteristics of the stock market sample data to determine which one of those distributions best fits each market. Most of the authors that have done similar research, firstly choose one or a couple of distributions, but in our observation we have used 56 distributions. We employed the Anderson-Darling criterion and accordingly we made a ranking of all the distributions, thus choosing the one that was a best fit according to this criterion.

This paper is organized as follows. Section 1 provides a review of the literature. In Section 2 we explain the data sample used in the analysis, and in Section 3 we present the method used in the analysis. Section 4 shows our empirical results. Finally, we provide a conclusion.

1. LITERATURE REVIEW

The assumption of normal distribution of the stock returns is incorporated in the most popular and most used models in the theory and practice of financial economics. Among them we will mention the mean-variance Markowitz Portfolio Theory (Markowitz 1952), CAPM (Sharpe 1964), and the Consumption CAPM (Lucas 1978). Additionally, the Black-Scholes option pricing model (Black and Scholes 1973; and Merton 1973) is derived based on the assumption that equity prices follow a geometric Brownian motion process, which has normally distributed increments. The bell-shaped normal distribution is completely characterized by two parameters, the mean and SD. The simple logic underlying that model is if return expectations implicit in asset prices are rational, then actual rates of return should be normally distributed around these expectations. In such

a way, investment management is far more tractable when rates of return can be well approximated by the normal distribution.

Contrary to theoretical assumptions prevalent in the theory of financial economics, the empirical evidence strictly rejects the normal distribution of the stock returns. Today there are many studies that confirm that the long horizon returns are often found to be approximately normally distributed, and over short horizons, equity returns are far from normal. Most of the studies show that returns on stocks display significant leptokurtosis, and in many cases, skewness (negative or positive depending on the period analyzed).

Among the first studies that found that the empirical distribution of the proceeds of the shares were not normal were Mandelbrot (1963, 1967) and Fama (1965). Mandelbrot (1963, 1967) presented evidence that distributions of returns can be well approximated by the stable Paretian distribution with a characteristic exponent less than 2 (a symmetric Levy stable law with tail index b about 1.7), thus exhibiting fat tails and an infinite variance. Fama, (1965) in his research on a sample of 30 stocks from DJIA Index, confirmed Mandelbrot (1963) that the stable Paretian distribution better characterized the stock price changes. Much later, Mittnik et al. (1998) confirmed these estimates of the power tail index, as well as Mantegna and Stanley (1995, 2000), who even suggested slightly different indices of the stable law ($b=1,4$).

Officer (1972) examined the validity of the symmetric stable class of distributions, and found that monthly returns follow normality, and the standard deviation appears to be a well behaved measure of scale. Clark (1973) found the lognormal distribution as a better fit on the sample of the data on cotton futures prices than a stable Paretian distribution proposed a couple of years previously by Mandelbrot (1963, 1967) and Fama (1965). Praetz (1972), analyzing weekly data samples from the Sydney Stock exchange, concluded that the Student-t distribution is a better fit than the stable Paretian because the Paretian distribution has an infinite variance property and unknown density function. Blattberg and Gonedes (1974) using a daily and weekly data sample of the DJI made a comparison of the three distributions – Student-t, normal and Cauchy, and concluded that the Student-t is the better fit than the normal on the sample of the daily returns, but normal distributions apply to the monthly returns. Akgiray and Booth (1987) also found that normal distribution is a good fit for the monthly stock returns. In this line is Hagerman (1978), who rejects the normal distribution and proposes that what should be used is a mix between the normal and the Student-t distribution as an alternative.

For describing security returns, Bookstaber and McDonald (1987) introduced the generalized distribution GB2, which is an extremely flexible distribution, containing a large number of well-known distributions, such as the lognormal, log-t, and log-Cauchy distributions, as special or limiting cases and allowing large, even infinitely higher moments. The properties of the GB2 make it useful in empirical estimation of security returns and in facilitating the development of option pricing models and other models that depend on the specification and mathematical manipulation of distributions.

Gray and French (1990) considered the distribution of log stock index returns of the S&P 500 and found that log stock return distributions do not follow the normal law as is often assumed, but instead have much longer tails and more peakedness than the normal family. Three alternative distributions: the scaled-t, logistic, and exponential power distributions, demonstrate a greater ability to model log stock index returns from the S&P 500 Composite Index. Of the three alternative models considered, the EPD appears to provide a superior fit.

Aparicio and Estrada (2001), using daily data of 13 European countries, made a comparison among four distributions: logistic, scaled-t, exponential power and a mixture of two Normal distributions. They found the scaled-t distribution as the most appropriate fit for their data sample and give partial support for a mixture of two Normal distributions. In addition, they note that normality may be a plausible assumption for monthly (but not for daily) stock returns. Normality was also not rejected for the weekly and monthly returns by Linden (2001). He used data samples of daily, weekly and monthly returns for the 20 most traded shares on the Helsinki Stock market. For the daily returns he found that the asymmetric Laplace is a better fit distribution than the Normal distribution.

The Harris and Kucukozmen (2001 and 2001a) model continuously compounded daily, weekly and monthly returns for the UK and US and Turkey, using two very flexible families of distributions exponential generalized beta (EGB) and generalized-t distribution (SGT). They found that both EGB and SGT distributions provide a substantial improvement over the normal distribution, while the SGT provides a marginally superior fit over the EGB. Their preferred distributions for daily equity returns are the skewed-t for the UK and the generalized-t for the US.

Malevergne et. al (2005) used daily data of DJIA, and very frequent data: 5-min returns of the Nasdaq Composite index and 1-min returns of the S&P500. They propose a parametric representation of the tail of the distributions of returns encompassing both a regularly varying distribution in one limit of the parameters and rapidly varying distributions of the class of the stretched-exponential (SE) and the log-Weibull or Stretched Log-Exponential (SLE) distributions in other limits. Using the method of nested hypothesis testing (Wilks' theorem), they conclude that both the SE distributions and Pareto distributions provide reliable descriptions of the data but are hardly distinguishable for sufficiently high thresholds.

Rachev et al. (2005) used a sample of daily returns for 382 USA stocks, and examined in the framework of two probability models - the homoskedastic independent, identical distributed model and the conditional heteroskedastic ARMA-GARCH model. They strongly reject the Gaussian hypothesis for both models. They also found out that the stable Paretian hypothesis better explains the tails and the central part of the return distribution.

Chalabi et al (2012) used the generalized lambda distribution (GLD) family as a flexible distribution with which to model financial data sets. Corlu et. al (2016) investigates the ability of five alternative distributions to represent the behavior of daily equity index returns over the period 1979–2014: the skewed Student-t distribution, the generalized lambda distribution, the Johnson system of distributions, the normal inverse Gaussian distribution, and the g-and-h distribution. They found that the generalized lambda distribution is a prominent alternative for modeling the behavior of daily equity index returns.

2. DATA

Our research encompasses 10 emerging countries from SEE. The raw data samples used in this paper are daily, weekly and monthly observations of their blue chip stock market indices. The countries and their indices are: Bosnia and Herzegovina (B&H) – SASX10,

Bulgaria (BUG) – SOFIX Index, Croatia (CRO) – CROBEX, Greece (GRE) - ATG, Macedonia (MAC) – MBI10, Montenegro (MNE) – MONEX20, Romania (ROM) – BET Index, Slovenia (SLO) – SBITOP, Serbia (SRB) – BELEX15, Turkey (TUR) – BIST30. Data for all 10 series were obtained from TR Datastream for a period of 5 years: 1 July, 2011 to 30 June, 2016 (except MNE to 31 March, 2015). This yield between 1215 to 1261 daily observations and the differences among the countries arise because of the non-working days in the country; 262 weekly and 60 monthly observations. The data are the index prices expressed in local currencies. This is the generally accepted approach in order to avoid the currency fluctuation effect.

We performed our analysis using continuously compounded stock market returns defined as $R_t = [\ln(I_t) - \ln(I_{t-1})]$, where R_t and I_t are the return and the index price in day t , respectively.

In tables 1, 2 and 3 we present relevant summarized information about the daily, weekly and monthly returns under consideration, respectively. The tables report the first four moments of each series, the minimum and maximum, the skewness and kurtosis, and also the Jarque-Bera statistic for normality.

The deviation from the normality can be gathered immediately considering the coefficients of skewness and kurtosis. As it is found in many other studies, it is also evident here that the daily stock returns represent leptokurtic distribution, thus exhibiting fat tails (and high peaks). This is a reflection of the occurrence of a number of large market movements in all markets, and they are not as extreme since in our analysis we skip the period of the greatest disturbances from the two crises (the Great financial crisis and the European debt crisis). The highest one-day rise on all markets averaged 6%, and the highest one-day decline on all markets averaged 7%. That on average represents 4 to 7 standard deviations above the mean, and 5 to 9 standard deviations below the mean, on the different markets.

The skewness also deviates from the normality. In all markets the coefficient of skewness of the daily returns is negative (except B&H and MNE). This negative skew indicates that the tail on the left side is longer and fatter than the right side. The longer and fatter the tail of a distribution is, the more extreme values it contains. Here we will refer to Peiro (1999), who notes that when the data is leptokurtic, then the skewness statistic is not valuable, since the coefficient of skewness gives an unrealistic picture. That is, by removing just two or three outliers in 1250 observations, it results in very large change of the coefficient of skewness.

We present the Jarque-Bera statistics as a formal testing for normality. This test should confirm if the distribution of the return is normal. The null hypothesis of this test is that the data follows a normal distribution. In all cases the probability is less than 1%, and we can clearly reject the null hypothesis and conclude that the distribution of the daily stock market returns is not normal.

The preliminary statistics of the weekly and monthly stock returns are presented in Table 2 and Table 3 respectively. In almost all cases, the distribution of weekly and monthly returns is leptokurtic. Five countries have negative and five positive skewness of the weekly returns. In the case of the monthly returns only two of the countries have positive skew. The Jarque-Bera statistics is significant in all cases for the weekly returns (except TUR). But in the case of the monthly returns it is not significant in four markets (CRO, GRE, SRB and TUR).

Table 1. Preliminary statistics of the distributions of daily stock returns

	B&H	BUG	CRO	GRE	MAC	MNE	ROM	SLO	SRB	TUR
Mean	-0.0003	0.0001	-0.0002	-0.0007	-0.0003	0.0000	0.0001	-0.0001	-0.0002	0.0002
Median	0.0000	0.0000	-0.0003	-0.0001	-0.0004	0.0002	0.0004	0.0000	-0.0002	0.0006
Maximum	0.0472	0.0564	0.0339	0.1343	0.0339	0.0693	0.0614	0.0342	0.0823	0.0691
Minimum	-0.0380	-0.0474	-0.0468	-0.1771	-0.0448	-0.0658	-0.0876	-0.0606	-0.0741	-0.1090
Std. Dev.	0.0076	0.0083	0.0062	0.0251	0.0065	0.0099	0.0100	0.0098	0.0083	0.0159
Skewness	0.0613	-0.1036	-0.4446	-0.3825	-0.1743	0.0809	-0.8204	-0.4870	-0.1715	-0.4097
Kurtosis	8.8753	7.4228	7.7462	8.1310	6.6125	10.968	12.3624	6.0769	19.3163	6.2705
Jarque-Bera	1797.1*	1010.4*	1209.5*	1362.4*	672.2*	2403.1*	4731.9*	540.3*	13993.9*	596.7*
N° of observations	1249	1237	1245	1215	1225	908	1257	1245	1261	1260

Note: * means significant at 1% level and ** means significant at 5% level

Table 2. Preliminary statistics of the distributions of weekly stock returns

	B&H	BUG	CRO	GRE	MAC	MNE	ROM	SLO	SRB	TUR
Mean	-0.0015	0.0004	-0.0011	-0.0032	-0.0015	0.0003	0.0007	-0.0004	-0.0009	0.0009
Median	-0.0010	0.0000	-0.0013	-0.0003	-0.0020	0.0004	0.0012	0.0001	0.0007	0.0025
Maximum	0.0497	0.0801	0.0446	0.1487	0.0656	0.0881	0.0681	0.0925	0.0978	0.0940
Minimum	-0.0508	-0.0593	-0.0725	-0.2254	-0.0500	-0.0878	-0.0825	-0.0630	-0.0594	-0.1038
Std. Dev.	0.0151	0.0189	0.0141	0.0550	0.0168	0.0234	0.0223	0.0220	0.0197	0.0344
Skewness	-0.3062	0.1700	-0.4250	-0.3768	0.4324	0.0139	-0.5243	0.2234	0.1172	-0.3193
Kurtosis	4.3420	4.7827	6.1445	3.7362	4.5682	4.9420	4.5024	4.1096	5.2742	3.2342
Jarque-Bera	23.75*	35.95*	115.83*	11.89*	35.01*	30.65*	36.09*	15.62*	57.06*	5.05
N° of observations	262	262	262	257	262	195	262	262	262	262

Note: * means significant at 1% level and ** means significant at 5% level

Table 3. Preliminary statistics of the distributions of monthly stock returns

	BIH	BUG	CRO	GRE	MAC	MNE	ROM	SLO	SRB	TUR
Mean	-0.0067	0.0016	-0.0048	-0.0143	-0.0070	0.0010	0.0027	-0.0014	-0.0035	0.0034
Median	-0.0037	0.0008	-0.0043	-0.0041	-0.0103	0.0018	0.0098	-0.0021	-0.0003	-0.0022
Maximum	0.0719	0.3567	0.0813	0.1985	0.1239	0.0856	0.1196	0.1571	0.0994	0.1404
Minimum	-0.1206	-0.4063	-0.0924	-0.2867	-0.0814	-0.2170	-0.1530	-0.1078	-0.1355	-0.1421
Std. Dev.	0.0307	0.0838	0.0313	0.1087	0.0410	0.0540	0.0499	0.0454	0.0502	0.0679
Skewness	-1.0433	-0.6125	-0.2493	-0.3522	1.0917	-1.2356	-0.7648	0.4931	-0.5054	-0.0175
Kurtosis	6.0702	15.5587	3.6255	2.6707	4.9806	7.0475	4.0877	4.5384	3.1834	2.3441
Jarque-Bera	34.45*	398.05*	1.60	1.51	21.72*	42.17*	8.81**	8.35**	2.64	1.08
N° of observations	60	60	60	60	60	45	60	60	60	60

Note: * means significant at 1% level and ** means significant at 5% level

3. METHOD

3.1. Tests of Goodness-of-fit

A Goodness-of-fit test is a procedure for determining whether a sample of n observations, x_1, \dots, x_n , can be considered as a sample from a given specified distribution. There are many tests developed for determining whether a sample could have arisen from a specific distribution, while the most popular are: the Kolmogorov-Smirnov (K-S), Anderson-Darling (A-D), and Chi-squared (χ^2 test). In addition, other well-known tests are: the Cramer-von Mises, Shapiro-Wilk (S-W), Hosmer-Lemeshow (H-L), Henze-Zirkler (H-Z) test.

Simply, with the Goodness-of-fit tests, we are measuring the "distance" between the observed data sample and the distribution we are testing (named *test statistic*), and then we make a comparison of that distance to some threshold value (named *critical value*). The fit can be considered a good fit only if the *test statistic* is smaller than the *critical value*. The logic of applying the various of the above-mentioned goodness-of-fit tests is the same, and they differ in the way of computing the test statistics and critical values.

The test statistics are usually defined as some function of sample data and the theoretical (fitted) cumulative distribution function. The critical values depend on the sample size and the significance level that is chosen. The significance level is the probability of rejecting a fitted distribution (as if it was a bad fit) when it is actually a good fit. The significance level that we are using here is $\alpha=0.05$. Since the goodness-of-fit test statistics indicate the distance between the data and the fitted distributions, it is obvious that the distribution with the lowest statistic value is the best fitting model. Here we take only the first ranked distribution according to the goodness-of-fit test.

Specifically, for the identical and independently distributed random sample X of size n from an unknown distribution, we use the test statistic $S = S(X|m)$ for testing the null hypothesis in the form H_0 : X follow the specified distribution. In a general sense, the test statistic S depends on the parameters θ of the distribution to be tested. Accordingly, $S = S(X|\theta, m)$ is a function of θ generally. H_0 can be tested for many of the distribution tests, but now we use the test statistic in the form $S(X|\theta_0, m)$, where H_0 : X follows the specified distribution with parameters θ_0 fixed. Now, for the fixed θ_0 , we can determine the exact distribution of the test statistic $S(X|\theta, m)$. For the test of H_0 to be exact we determine if the test has a correct type 1 error. Accordingly, we consider the significance level α meaning that the test incorrectly rejects the null hypothesis with probability α . This condition is equivalent to the p-value having a Uniform distribution under the null hypothesis.

3.2. Anderson-Darling tests of Goodness-of-fit

Of the many quantitative goodness-of-fit techniques we mostly prefer the Anderson-Darling test, which will be applied here. Engmann and Cousineau (2011) compare the K-S test and the A-D test, presenting conclusive evidence that the A-D test is more powerful. A-D test is a modification of the K-S test and it is more sensitive to deviations in the tails of the distribution than the older K-S test. Anderson and Darling (1952, 1954) proposed a test goodness-of-fit which can be used to determine if a specified sample of data came from a population with a specific distribution. They provided a modification of the Kolmogorov-Smirnov (K-S) test, giving more weight to the tails than the K-S test. The Anderson–Darling test belongs to the class of quadratic EDF statistics, tests based on the empirical distribution function (Stephens, 1986). The procedure of the A-D test implies a comparison of the fit of an observed cumulative distribution function to an expected cumulative distribution function. Let F be the hypothesized distribution, and F_n be the empirical (sample) cumulative distribution function, then the distance between F and F_n measured by the quadratic EDF statistics is

$$n \int_{-\infty}^{\infty} (F_n(x) - F(x))^2 w(x) dF(x) \quad (1)$$

where, the $w(x)$ is a weighting function. The Anderson–Darling (1954) test is based on the distance

$$A = n \int_{-\infty}^{\infty} \frac{(F_n(x) - F(x))^2}{F(x)(1 - F(x))} dF(x) \quad (2)$$

which is obtained when the weight function is $w(x) = [F(x)(1 - F(x))]^{-1}$, with which Anderson–Darling distance places more weight on observations in the tails of the distribution.

The Anderson-Darling test is defined as:

H_0 : The data follow a specified distribution

H_A : The data do not follow the specified distribution

With the A-D test we can assess whether the observed data sample x_1, \dots, x_n comes from some specified probability distribution. It utilized the fact that with a given hypothesized underlying distribution and assuming the data does arise from this distribution, the frequency of the data can be assumed to follow a Uniform distribution. Then, using the distance test (Shapiro, 1980), we can test the data for uniformity. The Anderson-Darling statistic (A^2) is defined as

$$A^2 = -n - S \quad (3)$$

$$S = \frac{1}{n} \sum_{i=1}^n (2i - 1) \cdot [\ln F(X_i) + \ln(1 - F(X_{n-i+1}))] \quad (4)$$

where $\{x_1 < \dots < x_n\}$ is the ordered (from smallest to largest element) sample of size n , and $F(X)$ is the underlying theoretical cumulative distribution to which the sample is compared. The null-hypothesis that $\{x_1 < \dots < x_n\}$ comes from the underlying distribution $F(X)$ is rejected if A^2 is greater than the critical value A_α at a given α (for a table of critical values for different sample sizes (Stephens 1974, 1976, 1977, 1979), (D'Agostino and Stephens 1986).

4. EMPIRICAL RESULTS

For the data samples on the stock market returns we are examining which theoretical probability distribution fits most. We examine 56 theoretical distributions and implement *goodness-of-fit tests* to select the best fitting distribution for our data. By applying the Anderson-Darling test at the significance level of $\alpha=0.05$, with a critical value of 2.5018, we calculated the test statistic for each distribution. Then we ordered the distributions according to the A-D test statistic from the lowest to the highest value of the test statistic. To the one with the smallest value of the test statistics we gave the rank of No. 1. Since the goodness-of-fit test statistics indicate the distance between the data and the fitted distributions, it is obvious that the distribution with the lowest statistic value is the best fitting model. Finally, the fit can be considered a good fit only if the test statistic is smaller than the critical value.

In Tables 4, 5 and 6 we present the most suitable probability distribution for each data sample. We consider it as the optimal distribution. Also, in each table, in the first panel we present the estimated parameters for the specified distribution, and in the second we present the estimated moments (mean, standard deviation, skewness, kurtosis, minimum and maximum). The general properties of these distributions are presented in Appendix A.

From Table 1, we can see that the daily stock market returns in almost all of the SEE emerging markets have a Johnson SU probability distribution, except for Bosnia and Herzegovina that have Laplace, and Slovenia that has a Hypersecant probability distribution. From Table 2, it is obvious that for the weekly returns there is a great divergence of the probability distributions among the countries. The same conclusion regards the monthly returns, see Table 3.

Table 4. Anderson-Darling test of Goodness-of-fit of the daily stock market returns

	B&H	BUG	CRO	GRE	MAC	MNE	ROM	SLO	SRB	TUR
Distribution	Laplace	Johnson SU	Johnson SU	Johnson SU	Johnson SU	Johnson SU	Johnson SU	Hypersecar	Johnson SU	Johnson SU
Parameters	$\lambda=185.4$ $\mu=-0.0003$	$\gamma=0.04491$ $\delta=1.4505$ $\lambda=0.00935$	$\gamma=0.19124$ $\delta=1.4477$ $\lambda=0.00685$ $\xi=0.000923$	$\gamma=0.15209$ $\delta=1.4128$ $\lambda=0.02682$ $\xi=0.00301$	$\gamma=0.08948$ $\delta=1.5327$ $\lambda=0.00791$ $\xi=0.00023$	$\gamma=0.02224$ $\delta=1.2672$ $\lambda=0.00894$ $\xi=0.000166$	$\gamma=0.22702$ $\delta=1.2573$ $\lambda=0.00871$ $\xi=0.00023$	$\sigma=0.00983$ $\mu=-0.000064$	$\gamma=0.0299$ $\delta=1.1108$ $\lambda=0.00583$ $\xi=0.000066$	$\gamma=0.23792$ $\delta=1.6039$ $\lambda=0.02045$ $\xi=0.00386$
Sample size	1249	1237	1245	1215	1225	908	1257	1245	1261	1260
Mean	-0.0003	0.0001	-0.0002	-0.0007	-0.0003	0.0000	0.0001	-0.0001	-0.0002	0.0002
St. deviation	0.0076	0.0083	0.0062	0.0251	0.0065	0.0099	0.0100	0.0098	0.0083	0.0159
Skewness	0	-0.0935	-0.4022	-0.3579	-0.1455	0.0889	-0.9464	0	-0.2457	-0.3184
Kurtosis	3	4.4508	4.7663	5.1552	3.6340	8.0199	9.3982	2	16.3980	3.2883
Min	-INF	-INF	-INF	-INF	-INF	-INF	-INF	-INF	-INF	-INF
Max	+INF	+INF	+INF	+INF	+INF	+INF	+INF	+INF	+INF	+INF

Table 5. Anderson-Darling test of Goodness-of-fit of the weekly stock market returns

	B&H	BUG	CRO	GRE	MAC	MNE	ROM	SLO	SRB	TUR
Distribution	Laplace	Error	Hypersecant	Burr (4P)	Hypersecant	Error	Dagum (4P)	Error	Dagum (4P)	Johnson SU
Parameters	$\lambda=93.471$ $\mu=0.00149$	$k=1.1802$ $\sigma=0.01892$ $\mu=4.0305E-4$	$\sigma=0.01414$ $\mu=0.0011$	$k=2.6643$ $\alpha=17.221$ $\beta=0.72387$ $\gamma=-0.6744$	$\sigma=0.01682$ $\mu=0.00155$	$k=1.1438$ $\sigma=0.023$ $\mu=2.5E-4$	$k=0.67358$ $\alpha=271.49$ $\beta=2.7643$ $\gamma=-2.7564$	$k=1.3544$ $\sigma=0.0215$ $\mu=-3.5E-4$	$k=0.40191$ $\alpha=14.291$ $\beta=0.11038$ $\gamma=0.0997$	$\gamma=4.1025$ $\delta=5.8207$ $\lambda=0.15611$ $\xi=0.12205$
Sample size	262	262	262	257	262	195	262	262	262	262
Mean	-0.0015	0.0004	-0.0011	-0.0032	-0.0016	0.0003	0.0006	-0.0004	-0.0016	0.0009
St. deviation	0.0151	0.0189	0.0141	0.0550	0.0168	0.0235	0.0219	0.0220	0.0188	0.0344
Skewness	0	0	0	-0.37428	0	0	-0.43319	0	-0.14656	-0.07065
Kurtosis	3	1.8375	2	0.69457	2	2.0241	1.6069	1.1543	0.88877	0.26168
Min	-INF	-INF	-INF	-0.67449	-INF	-INF	-2.7564	-INF	-0.0997	-INF
Max	+INF	+INF	+INF	+INF	+INF	+INF	+INF	+INF	+INF	+INF

Table 6. Anderson-Darling test of Goodness-of-fit of the monthly stock market returns

	B&H	BUG	CRO	GRE	MAC	MNE	ROM	SLO	SRB	TUR
Distribution	Dagum (4P)	Cauchy	Burr (4P)	Johnson SB	Burr (4P)	Log-Logistic (3P)	Burr (4P)	Error	Dagum (4P)	Johnson SB
Parameters	$\alpha=9.0505E+5$ $\beta=10569.0$ $\gamma=10569.0$	$\sigma=0.02451$ $\mu=-7.17E-4$	$k=2.0286$ $\alpha=21.53$ $\beta=0.46951$ $\gamma=0.45363$	$\gamma=0.9743$ $\delta=1.6923$ $\lambda=0.44884$ $\xi=0.5468$	$k=0.81162$ $\alpha=10.388$ $\beta=0.19015$ $\gamma=0.20742$	$\alpha=5.4631E+8$ $\beta=1.5499E+7$ $\gamma=1.5499E+7$	$k=2.4702$ $\alpha=10567.0$ $\beta=345.74$ $\gamma=345.74$	$k=1.1918$ $\sigma=0.04541$ $\mu=0.00138$	$k=0.2594$ $\alpha=14.553$ $\beta=0.24625$ $\gamma=0.20468$	$\gamma=0.03668$ $\delta=1.3842$ $\lambda=0.41859$ $\xi=0.20834$
Sample size	60	60	60	60	60	45	60	60	60	60
Mean	-0.0068	-0.0007*	-0.0047	/	-0.0069	0.0029	0.0033	-0.0014	-0.0038	/
St. deviation	0.0288	/	0.0309	/	0.0400	0.2310	0.0479	0.0454	0.0494	/
Skewness	216.15	/	-0.28634	/	1.3552	-1.01E+08	-0.68849	0	-0.33879	/
Kurtosis	-67740000	/	0.78154	/	6.1008	9.01E+15	-0.0388	1.7819	0.35862	/
Min	-10569	/	-0.45363	-0.5468	-0.20742	-1.55E+07	-345.7	-INF	-0.20468	-0.20834
Max	+INF	/	+INF	0.29808	+INF	+INF	+INF	+INF	+INF	0.21025

*Mode

CONCLUSION

The most popular models in the theory and applications of financial economics, as are the CAPM, Black-Sholes, Markowitz portfolio theory and others, implicitly or explicitly are based on the assumption of the normal distribution. Today there is considerable empirical evidence that the normal distribution is not the best fit distribution of the stock market returns. This conclusion is especially emphasized in the case of the daily returns, where some authors still do not reject it for the weekly and monthly stock returns.

In this paper we have provided a search of the best fit distribution for the daily, weekly and monthly stock market returns in the case of 10 emerging Southeast European markets: Bosnia and Herzegovina, Bulgaria, Croatia, Greece, Macedonia, Montenegro, Romania, Slovenia, Serbia, and Turkey. After calculation of their basic statistic parameters, we provide a formal testing for the normality. We clearly rejected the normality, especially for the case of the daily returns. We then attempted to find the specification that best fits the data in each market. We didn't specify in advance any distribution to be tested for its validity as many authors do, nor did we use any underlying financial theory. We took all distributions and all of them were subject to testing. We employed the Anderson-Darling methodology for computing the test statistic of each distribution. Then we made a ranking of all those distributions. The best distribution, ranked as first, was that with the lowest test statistic. Finally, if its *test statistic* is smaller than the *critical value* for the level of confidence $\alpha=0.05$ we choose that distribution as the most optimal for the data sample.

The most optimal distribution for the daily stock return is Johnson SU distribution. It is the best fitted distribution in eight SEE markets (except B&H with Laplace and SLO with Hypersecant). There is not one predominant distribution for the weekly stock returns for all cases, there are even six distributions that have appeared as most optimal for the different countries: Burr (4P), Dagum (4P), Error, Hypersecant, Johnson SU and Laplace. The same is the situation with the monthly returns, where six distributions also appear as best fit: Burr (4P), Cauchy, Dagum (4P), Error, Johnson SB, Log-Logistic (3P).

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