A Fuzzy Expert System to Solve Multiobjective Optimization Problems

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Abstract

Due to the human nature, the Decision Maker (DM) may get tired or bored as a result of tedious interaction with the solution procedure of a multiobjective mathematical model. Consequently, the accuracy of information obtained from him/her or the selection of an appropriate efficient solution will be affected. Therefore, this paper presents an interactive approach for solving Multiobjective Decision Making (MODM) problems in the presence of fuzzy preferences in addition of architecture of a fuzzy expert system. In the solution method, the DM's preferences are represented by using linguistic variables. The suitable fuzzy membership function will be selected, which has the ability to obtain an acceptable efficient solution according to the DM preferences. The main objectives of this approach are minimizing the interaction time and selecting an appropriate efficient solution that fits the DM's preferences.

Keywords: Multiobjective decision making; Linguistic variables; Membership functions; Fuzzy preferences.

1.Introduction

During the last three decades, several methods have been developed to solve MODM problems. These methods can be classified into the following main approaches:

1. Goal programming [1],
2. Utility approaches [2,3],
3. Interactive procedures [4], and
4. Fuzzy programming [5].

Among the solution approaches, the interactive methods are promising and thus becoming popular for solving MODM problems. A comparative study [6] has shown the superiority of the interactive methods over the other approaches for

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solving MODM problems. While most of the MODM methods used one of the above four approaches, some methods combine more than one approach, i.e. some MODM methods used a combination of two approaches. For example: elicitation of some preference information prior to the use of an algorithm which employs a progressive articulation of preferences may allow the algorithm to operate more efficiently [7].

Many interactive approaches were developed to solve MODM problems in fuzzy environment, such as, Werners' method [8] which applied Bellman and Zadah max-min operator [9] to transfer the model into a symmetric model and solve it by using Zimmermann's fuzzy programming approach [10]. This method considers that the DM has the capability to construct and modify the membership function for each objective and constraint. Tapia and Murtagh [11] suggested an approach to solve linear and nonlinear MODM problems. This approach considers that the DM has selection criteria (preference selection) of the best compromise solution and non-negative underachievement that he/she is willing to accept for each objective function. Also, he/she can identify a finite number of efficient solutions, which satisfy his/her implicit preference structure. Leung [12] suggested a method to solve a hierarchical fuzzy objective problem by applying the preemptive fuzzy goal programming approach. In this method, the DM is asked to prioritize the objective functions to start finding the optimum solution of the objective function with the highest priority, under a set of constraints, then finding the optimum solution of the next important objective function with a tolerance trade-off specified for the optimal value of the preceding objective and so on. A new approach to solve MODM problems with imprecise objective and constraints' coefficients that have trapezoidal membership functions has been introduced in [13]. Also, W. Abd El-Wahed [14] used Zimmermann's fuzzy programming approach to study the solution behavior of the multiobjective transportation problem under fuzziness. On the other side, the fuzzy linguistic approach has been applied successfully in the area of decision making [15,16,17], in case that the decision maker(s) can not assess in a quantitative form, but rather in qualitative one, i.e. with imprecise or vague knowledge.

On the other hand, fuzzy expert system (FES) has been successfully applied to a wide range of problems from different areas presenting ambiguity and vagueness in different ways [27, 28, 29].

Due to the ability of FES to deal with uncertainty and vagueness; we will focus our attention on coupling symbolical and numerical processing, considering the conflict among objectives in one approach, to get an effective way to generate high quality estimates with small computational effort.

In this paper, we present an interactive approach to solve the multiobjective problems based on linguistic variables and the achievement level of each objective function that represent the DM's preferences. The suggested approach converts the DM's preferences into their corresponding achievement intervals and the mathematical model into a single objective model using these intervals to find the preferred solution that fits his/her preferences.

This paper is structured as follows: Section 2 presents some basic notations. In Section 3 the choice of linguistic variables is given. Section 4 presents the mathematical analysis of the given system. Section 5 presents the problem formulation. In Sections 6 and 7 the proposed interactive approach is described. Section 9 presents the suggested architecture of the fuzzy expert system Section 9 presents an illustrative example. Finally, in Section 10 some concluding remarks are made.

2. Basic Notations:

The following definitions represent the core of this study:

Definition 1: (*Non-Dominated Solution*) [18]
A nondominated solution is one in which no one objective function can be improved without a simultaneous detriment to at least one of the other objectives of the MODM problems. That is, $\underline{x}^*$ is a nondominated solution to the vector maximization problem iff there does not exist any $\underline{x} \in X$ such that $f_i(\underline{x}^*) \leq f_i(\underline{x})$ for all i and $f_j(\underline{x}^*) < f_j(\underline{x})$ for at least one j.

Definition 2: (*Preferred Solution*) [18]
The preferred solution is a nondominated solution, which is chosen by the DM, through some additional criteria, as a final decision. As such, it lies in the region of acceptance of all the criteria values for the problem. It is also known as the best solution.

Definition 3: (*Linguistic Variables*) [19]
A linguistic variable is fully characterized by a quintuple (v, T, X, g, m) in which v is the name of the variable, T is the set of linguistic terms of v that refer to a base variable whose values range over a universal set X, g is a syntactic rule (which usually has the form of a grammar) for generating linguistic terms, and m is a semantic rule that assigns to each linguistic term $t \in T$ its meaning, and $m(t)$ is a fuzzy set on X (i.e. $m: T \to F(x)$).

Definition 4: (*Fuzzy Maximizing Set*) [20]
Let f be a real-value function in x. Let f be bounded from below by $inf(f)$ and from above by $sup(f)$. The fuzzy set $\tilde{A}=\{x, \mu_{\tilde{A}}(x)\}$ with

$$\mu_{\tilde{A}}(x) = \frac{f(x) - \inf(f)}{\sup(f) - \inf(f)} \qquad (1)$$

is called the fuzzy maximizing set. Where $\mu_{\tilde{A}}(x) \neq 1$ is the estimates of how much x is far from being a maximum x_o (which is such that $\mu_{\tilde{A}}(x_o)=1$)

Definition 5: (*Trapezoidal Fuzzy Number*) [21]

A trapezoidal fuzzy number $\tilde{A}$ can be denoted by a (a_1, a_2, a_3, a_4) and its membership function is defined as in (2)

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & if \quad x<a_1 \quad or \quad x>a_4 \\ \dfrac{x-a_1}{a_2-a_1} & if \quad a_1 \le x < a_2 \\ 1 & if \quad a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & if \quad a_3 < x \le a_4 \end{cases} \qquad (2)$$

If $a_2 = a_3$, then $\tilde{A}$ is reduced to a triangular fuzzy number.

If $a_1 = a_2 = a_3 = a_4$, then $\tilde{A}$ is reduced to a real number. The graphical representation of the trapezoidal membership function is shown in Figure (1).

Definition 6: (*Interval-Value Fuzzy Number*) [22]

Interval- value fuzzy number can be defined as follows:

$I = [0, 1]$, for every $[I] \subseteq I$, $[I] = \{[a, b] \mid a \le b;\ a, b \in I\}$.

3. The Choice of Linguistic Variable

The main aim of using linguistic variables is to supply the DM with a few words by which he/she can express his/her preferences. In order to achieve such goal, it is important to analyze the cardinality of the linguistic term-set used to express the information [23]. Also, Miller [24] pointed out that "seven plus or mines two" represents the greatest amount of information an observer can give about the objects on the basis of absolute judgment. Herrare [23,25] used 3 to 9 linguistic terms to capture the DM's preferences to evaluate a set of alternatives. Since the DM is the person who knows much more about his/her problem, the number of linguistic terms that can be used to interact with the DM can be concluded from his/her words [26], as an example, if the DM uses word "medium, very high" that means the maximum number of linguistic terms included in the membership function (MF) is 5. Another example; if the DM uses the word "perfect, medium" in his/her preferences that means the suitable number of linguistic terms is 7. In addition, most of the authors see that the trapezoidal MF is a very suitable MF to represent the linguistic preferences of the DM [23,25].

4. Mathematical Analysis of the Given System

In this section, we will highlight and prove some fuzzy and mathematical theories that will serve our goals. These theories were proved based on definitions that were introduced in the previous section.

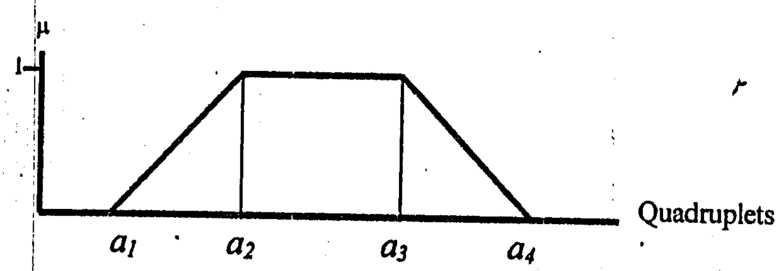

Figure 1. .Trapezoidal membership function

Theorem 1. [5]

There exists one and only one linear transformation $L: x \rightarrow \alpha x + \beta$, which transfers two different points, x_1 and x_2 of one membership function into two different points of anther membership function, y_1 and y_2. Where:

$$\alpha = \frac{y_2 - y_1}{x_2 - x_1} \quad , \quad \beta = \frac{y_1 x_2 - x_1 y_2}{x_2 - x_1} \qquad (3)$$

Theorem 2.

Let λ be the achievement level of the DM, $\lambda \in I$ and $\tilde{A}_k = (a_1, a_2, a_3, a_4)$, then there exists an interval $[I] = [b_1, b_2]$ that represents the DM preferences for each objective function f_j and $a_1 \leq b_1 \leq a_2$, $a_3 \leq b_2 \leq a_4$.

Where $\tilde{A}_k = (a_1, a_2, a_3, a_4)\ \forall\ k = 1, 2, \ldots, K$ is a fuzzy number corresponding to the DM's linguistic term, which represents his preferences, and K is the number of linguistic terms of each objective function f_j, i.e., it is the number of linguistic terms of the corresponding membership function.

Proof

As illustrated in Figure 2 and from the basic mathematics.

$$\therefore \quad b_1 = \lambda (a_2 - a_1) + a_1 \qquad (4)$$

and

$$\therefore \quad b_2 = \lambda (a_3 - a_4) + a_4. \qquad (5)$$

From equations (4) and (5) we can find an interval $[b_1, b_2] \subseteq I$ using λ and the linguistic term $\tilde{A}_k = (a_1, a_2, a_3, a_4)$.

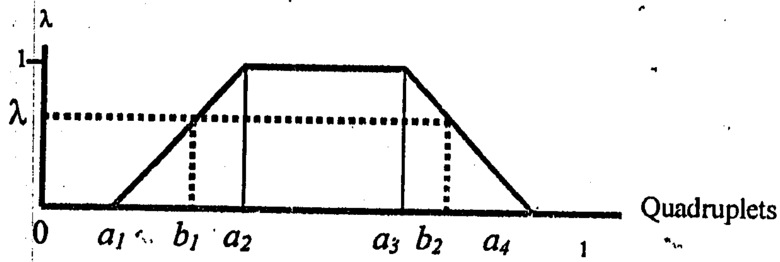

Figure 2. Acceptable interval of a linguistic term

Note :

If $\lambda = 0$ then $b_1 = a_1$ & $b_2 = a_4$ i.e. $[b_1, b_2] = [a_1, a_4]$ (6)

If $\lambda = 1$ then $b_1 = a_2$ & $b_2 = a_3$ i.e. $[b_1, b_2] = [a_2, a_3]$ (7)

From equations 6 and 7, then $b_1 \in [a_1, a_2]$, $b_2 \in [a_3, a_4]$ and the corresponding linear transformation can be written as follows:

$$\therefore L(\lambda) = \begin{cases} [a_1, a_4] & if \quad \lambda = 0 \\ [(a_2 - a_1)\lambda + a_1, (a_3 - a_4)\lambda + a_4] & if \quad 0 < \lambda < 1 \\ [a_2, a_3] & if \quad \lambda = 1 \end{cases} \tag{8}$$

So, the interval-value fuzzy number $L(\lambda)$ can be used to represent the DM's preferences instead of the linguistic variables $\tilde{A}_k$ at certain achievement level λ.

Lemma 1

If $[I] \subseteq [0,1]$ represents the DM's preferences for objective function f_i and value of $f_j(x_i)$ is a monotonic function, then we have

$T : [0,1] \rightarrow [f_l, f_u]$.

Where $f_l = min\, f_j(x), f_u = max\, f_j(x)$,

Result 1

If $[I] = [b_1, b_2] \subset [0,1]$, represents the DM's preferences about function f_j and f_j is a monotonic function then we can find $T : [I] = [b_1, b_2] \rightarrow [l, u]$.

Where:

$l = (f_u - f_l)\, b_1 + f_l$ and $u = (f_u - f_l)\, b_2 + f_l$

Proof

Since $f_l = min\, f_j(x) \le f(x_1) < f(x_2) \ldots < f(x_n) \le f_u = max\, f_j(x)$, then f is a monotonically increasing function, then there is a linear transformation from any interval to it.

By using theorem (1) and by replacing x, x_1, x_2, y, y_1 and y_2 by b_1, 0, 1, l, f_l, and f_u equation (1) will be

$$\alpha = \frac{f_u - f_l}{1 - 0} \quad , \quad \beta = \frac{f_l}{1 - 0}$$

$$\therefore \quad l = \alpha\, b_1 + \beta$$

$$\therefore \quad l = (f_u - f_l)\, b_1 + f_l \tag{9}$$

The same applies for finding u

$$\therefore \quad u = (f_u - f_l)\, b_2 + f_l \tag{10}$$

So, from equations (9) and (10) we can transform $b_1, b_2 \in [I]$ into their equivalent l, u; i.e., that $T([I]) = r$, $r = [l, u]$ and

$l = (f_u - f_l)\, b_1 + f_l$ *and* $u = (f_u - f_l)\, b_2 + f_l$

From lemma (1) and result (1), the DM's preferences for his/her achievement level λ can be represented mathematically according to the following theorem.

Theorem 3.

For every achievement level λ and linguistic term $\tilde{A}_k$ = (a_1, a_2, a_3, a_4) of any objective function f_j, there is an interval $[l, u] \subset [f_l, f_u]$ that represents the DM's preferences. That means, there is an interval $[l, u]$ that represents the DM's preferences instead of the linguistic term and the achievement level λ.

Where: $l = (f_u - f_l)\,[\lambda\,(a_2 - a_1) + a_1] + f_l$, and

$$u = (f_u - f_l)\,[\lambda\,(a_3 - a_4) + a_4] + f_l$$

Proof

The theorem can be easily proved as follows:

From result 1, and equations (9) and (10) we get

$l = (f_u - f_l)\, b_1 + f_l$ and $u = (f_u - f_l)\, b_2 + f_l$ (11)

Replacing b_1, b_2 by their values from Theorem 2, in equations (3) and (4) we will find that

$$l = (f_u - f_l)\,[\lambda\,(a_2 - a_1) + a_1] + f_l \quad (12)$$

$$u = (f_u - f_l)\,[\lambda\,(a_3 - a_4) + a_4] + f_l \quad (13)$$

The values of l and u equations (12) and (13) can be used to represent the DM's preference ($\tilde{A}_k$ and λ) concerning the objective function f_j.

Based on definition 4, the following result can be stated:

Result (2):

Let f be a real-value function in χ. and $\gamma = [l, u]$. Let f be bounded from below by $inf(f)$ and from above by $sup(f)$ and $\gamma = [l, u] \subseteq [inf(f), sup(f)]$.

Then the membership function $\mu_\gamma(\chi)$ which

$$\mu_\gamma(\chi) = \frac{f(\chi) - \inf(f)}{\sup(f) - \inf(f)}$$

maximizes f_j can be defined as follows:

Where: $inf(f) = l$ and $sup(f) = u$

Proof

Where f_j is monotonically (increase or decrease) and bounded from blow by $min\ f_j$ and bounded from above by $max\ f_j$. Then f_j is bounded over the period $[min\ f_j, max\ f_j]$.

Where $f(x)$ is monotonic function so for every $y = [l, u] \subseteq [min\ f_j, max\ f_j]$ is bounded from blow by $f(l)$ and from above by $f(u)$ over the region y. By using definition (4) then the fuzzy set $\tilde{A} = \{y, \mu_{\tilde{A}}(y)\}$ with

$$\mu_y(x) = \frac{f(x) - l}{u - l}$$

is the fuzzy maximized set of y.

5. Problem Formulation

The purpose of a MODM problem is to optimize J different objective functions subject to a set of constraints. A mathematical formulation of the MODM problem known as the Vector Maximization Problem (VMP) is

P1 : Max $f_j(C_k, x)$, $j = 1, 2, \ldots, J$

S.t. $x \in X = \{x = R^n \mid g_i(A_i, x)\{\leq, =, \geq\}b_i,\ i = 1, 2, \ldots, m,\ x \geq 0\}$,

Where:

x is an n-dimensional decision vector,

$C_k = (c_{k1}, c_{k2}, \ldots, c_{kn})$,

$A_i = (a_{i1}, a_{i2}, \ldots, a_{in})$,

$b_i = (b_{i1}, b_{i2}, \ldots, b_{in})^T$,

X is the decision space, and

$f_j()$ is a vector of j real-valued functions

In order to make the problem non-trivial; it is assumed that the objectives are in conflict and incommensurable. Zimmermann [10] suggested a solution method to solve such problems by converting the problem *P1* into a single objective function model *P2* by using the following steps:

1. Solve each objective function individually under the set of constraints,
2. Construct the payoff table,
3. Find the Minimum f_j^{min} and the maximum f_j^{max} for each objective function, and
4. Formulate the model P2 by substituting the values f_j^{min} and f_j^{max} in equation (15).

$$\mu_j(f_j(x)) = \begin{cases} 1 & f_j(x) \geq f_j^{max} \\ \dfrac{f_j(x) - f_j^{min}}{f_j^{max} - f_j^{min}} & f_j^{min} \leq f_j(x) \leq f_j^{max} \\ 0 & f_j(x) \leq f_j^{min} \end{cases} \qquad (15)$$

$$\begin{aligned} \textit{P2}: \ & \text{Max } \beta \\ & \text{s.t. } \beta \leq \mu_j(f_j(x)), \quad j = 1, 2, ..k. \\ & x \in X = \{x = R^n \mid g_i(A_i, x)\{\leq, =, \geq\} b_i, \ i = 1,2,\ldots, m, x \geq 0\}, \\ & 0 \leq \beta \leq 1 \end{aligned} \qquad (16)$$

6.The Suggested Approach

The accuracy of information obtained from the DM and his/her right selection of an efficient solution can be affected by the long period of interaction with a mathematical algorithm. Meanwhile, the selected solution may not be the best one that matches his/her preferences. So, in this paper we discuss an interactive approach for solving MODM problems in a fuzzy environment based on the DM's linguistic preferences and using the concept of the bounded objective method [18]. We assume that the DM has a fuzzy membership function to represent his/her degree of achievement λ_j for each objective function ∀j as illustrated in Figure (3). The interaction with the DM will be done through the base linguistic term (such as, Low, Medium, High) achievement for each objective function. The approach defines a function T to transfer the linguistic term $\tilde{A}_k$ with an achievement level λ_j that represents the DM preference to the corresponding achievement interval $[L_j, U_j]$ of each objective function f_j as follows:

In case of $max\, f_j(x)$:

$$L_j = (f_j^{max} - f_j^{min})\ [\lambda_j (a_2 - a_1) + a_1] + f_j^{min} \quad (17)$$

$$U_j = (f_j^{max} - f_j^{min})\ [\lambda_j (a_3 - a_4) + a_4] + f_j^{min} \quad (18)$$

In case of $min\, f_j(x)$:

$$L_j = f_j^{max} - (f_j^{max} - f_j^{min})\ [\lambda_j (a_3 - a_4) + a_4] \quad (19)$$

$$U_j = f_j^{max} - (f_j^{max} - f_j^{min})\ [\lambda_j (a_2 - a_1) + a_1] \quad (20)$$

Where:

λ_j is the degree of achievement of objective function f_j, (a_1, a_2, a_3, a_4) is a quadruplet of the linguistic variable that represents the DM's preferences of f_j, L_j and U_j are the lower and upper bounds that can be used to represent the acceptable range of achievement for each objective function, f_j^{min} and f_j^{max} are the minimum and maximum values of each objective function from the payoff table.

So, we will use equations (17-20) to find the achievement intervals that represent the DM preferences and replace equation (15) with equation (21) in the model *P2*.

$$\mu_j(f_j(x)) = \begin{cases} 1 & f_j(x) \geq U_j \\ \dfrac{f_j(x) - L_j}{U_j - L_j} & L_j \leq f_j(x) \leq U_j \\ 0 & f_j(x) \leq L_j \end{cases} \quad (21)$$

We can reduce the constraint (20) in model *P2* into the following form

$$\beta(U_j - L_j) \leq (U_j - f_j(x))$$

$$\beta(U_j - L_j) + f_j(x) \leq (U_j) \quad (22)$$

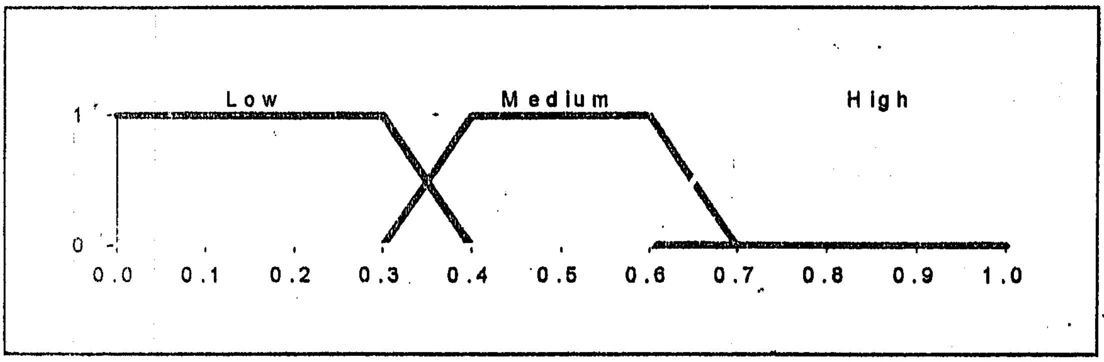

Figure 3 Degree of Achievement Fuzzy Membership Function

7. Proposed Interactive Approach

The suggested approach to solve the MODM problem based on the fuzzy preferences of the decision maker and using the concept of fuzzy sets, can be summarized as follows:

Step 1: Max $f_j(x)$, $j=1,2,\ldots,J$

S.T. $x \in X = \{x = R^n \mid g_i(A_i, x)\{\leq, =, \geq\}\ b_i,\ i=1,2,\ldots,m,\ x\geq 0\}$,

let x_j^* ($j=1,2,..K$) be the optimal solution to the objective function $f_j(x)$.

Take Max $f_j(x_j^*) = f_j^{max}$ $\forall j$.

Step2: Construct the payoff table by calculating the value of each objective at each optimal point $f_j(x_j^*)$.

Step 3: Find Min $f_j(x_j^*) = f_j^{min}$ $\forall j$ from the payoff table.

Step 4: Interact with the DM to discover his/her preferences in a linguistic variable form corresponding to each objective function.

Step 5: Transfer the linguistic variables (preferences) for each objective function into their corresponding lower bound L_j and upper bound U_j using its corresponding degree of achievement FM function and the f_j^{min} and the f_j^{max} $\forall j$.

Step 6: Formulate the single objective function model P2.

Step 7: Solve P2.

Step 8: Represent the solution to the DM. If he/she is satisfied go to Step 9, otherwise go to Step 4.

Step 9: Stop.

8. The FESGP Architecture

The role of FESGP is first, to interact with DM to define the suitable number of linguistic terms to be included in MF based on the natural language that DM uses. Then, the system calculates the acceptable interval [U_j, L_j] of each objective function based on the type of the objective function, DM preferences and the number of linguistic terms that he/she uses. Consequently, FESGP will support DM for the following: (1) Input data; (2) Specifying maximal and minimal bounds for each objective function; (3) Determining the aspiration level for each goal; (4) Constructing MF; and (5) Constructing the GP model. In order for FESGP to satisfy these requirements, it must contain the following five major parts in its architecture: User interface, Inference engine, Fuzzy knowledge base, Linear programming package, and GP package. We will describe the first three parts in greater detail.

8.1. User Interface

This part of the program contains the interactive questions that are divided into two parts, the first part contains the questions that deal with the problem structure; and the second part contains the questions concerning DM's linguistic preference.

8.2. Fuzzy Knowledge Base

Knowledge-based systems (KBS) are computer systems designed to replicate functions performed by a human being. KBS can perform such tasks as selecting the right grade and kind of material for customers, diagnosing equipment failures, and scheduling machines on the factory floor. Other similar systems exist which give advice in auditing, sales management, media selection, configuring computer systems, assigning rooms to guests at hotels and decision making [29]. The fuzzy knowledge base can be represented by different forms such as; frames, semantic networks or production rules which are commonly used. As presented in previous sections, the logical rules can be divided into two sets: rules to determine the number of linguistic terms, and upper and lower bound rules. These sets of rules can be described as follows:

First: Linguistic Terms Rules:

This set of rules is concerned with defining the number of linguistic terms that fit DM's linguistic preferences described previously. This set of rules can be written as follows:

Rule (1)

IF DM Preference is Perfect *OR*
DM Preference is Don't Care
THEN
No-of-Linguistic-Terms is 7

Rule (2)

IF (DM Preference is Very High *OR*
DM Preference is Very Low) *AND*
(DM Preference is not Perfect
OR DM Preference is not Don't Care)
THEN
No-of-Linguistic-Terms is 5

and so on.

Second: Upper and Lower Bounds Rules:

This set of rules is concerned with calculating the intervals that represent DM's preference, i.e., the upper and lower bounds of each objective function according to DM's linguistic preferences, the objective function type and the number of linguistic terms used. Considering $d_j = F_j^{Max} - F_j^{Min}$, this set of rules can be written as follows:

Rule (1)

IF The Objective Function$_j$ Type is Maximization
AND DM Preference$_j$ is Perfect
AND Number of Linguistic Term is 7
THEN
$L_j = d_j [0.07\,\lambda + 0.85] + F_j^{Min}$
AND $U_j = F_j^{Max}$

Rule (2)

IF The Objective Function$_j$ Type is Maximization
AND DM-Preference$_j$ is Very High
AND Number of Linguistic Term is 5
THEN
$L_j = d_j [0.12\,\lambda + 0.88] + F_j^{Min}$ *AND*
$U_j = F_j^{Max}$

and so on.

8.3. Inference Engine

The inference engine is the process of deriving new information from other known information used in an expert system (ES). It contains mechanisms, strategies and control used to manipulate and apply knowledge to the problem. FESGP uses inference engine with backward chaining existing in the VP-Expert shell. VP-Expert can use backward and forward chaining.

9. Illustrative Example

To illustrate the approach, consider the following nutrition problem as an example of multiobjective linear programming problem [18]. The nutrition problem is to find the quantities of certain foods to meet nutritional requirements the minimum daily requirement of vitamin A and iron; and the balanced daily requirement of food energy and protein; and seek the following objective functions (1) Minimum cost, (2) Minimum cholesterol daily intake, and (3) Minimum carbohydrate daily intake.

Problem formulation

$$Min\, f_1(x) = 0.225\, x_1 + 2.2\, x_2 + 0.8\, x_3 + 0.1\, x_4 + 0.05\, x_5 + 0.26\, x_6$$
$$Min\, f_2(x) = 10\, x_1 + 20\, x_2 + 120\, x_3$$
$$Min\, f_3(x) = 24\, x_1 + 27\, x_2 + 15\, x_4 + 1.1\, x_5 + 52\, x_6$$

s.t.

$$720\, x_1 + 107\, x_2 + 7080\, x_3 + 134\, x_5 + 1000\, x_6 \geq 5000,$$
$$0.2\, x_1 + 10.1\, x_2 + 13.2\, x_3 + 0.75\, x_4 + 0.15\, x_5 + 1.2\, x_6 \geq 12.5,$$
$$344\, x_1 + 460\, x_2 + 1040\, x_3 + 75\, x_4 + 17.4\, x_5 + 240\, x_6 \geq 2500,$$
$$18\, x_1 + 151\, x_2 + 78\, x_3 + 2.5\, x_4 + 0.2\, x_5 + 4\, x_6 \geq 63,$$
$$x_1 \leq 6.0,\ x_2 \leq 1.0,\ x_3 \leq 0.25,\ x_4 \leq 10.0,\ x_5 \leq 10.0,\ x_6 \leq 4.0,$$
$$x_i \geq 0\ \forall\, i = 1, 2, ..6.$$

By solving each objective function separately and calculating the other objective functions at the solution point we can construct Table (1). From this table we can find the minimum and the maximum values of each objective function. After that using Table (1) and the degree of the achievement fuzzy membership function Figure (4). We can formulate the membership functions of all the objective functions.

From the payoff table we found that:

$2.25556 \leq f_1 \leq 3.95044$,
$17.9070 \leq f_2 \leq 96.6860$, and
$150.0464 \leq f_3 \leq 411.97680$.

Present the payoff table to the DM. Assume the DM represents his/her preferences according to the trapezoidal membership function shown in Figure (3) and the linguistic variables used to interact with the DM ($\tilde{A}_k$, λ) were (medium, 70%), (high, 80%), and (low, 70%) regarding the improvement of each objective functions f_1, f_2, and f_3 respectively. By using the payoff table the membership function can be formulated and we can determine the corresponding achievement interval of each linguistic variable concerning the objective functions f_1, f_2, and f_3 respectively. So, by using fuzzy linguistic terms of fuzzy membership functions, their achievement level, and equations (17-20) to determine the values of (U_j and L_j) for each objective function, we found that (L_1= 2.88267, U_1=3.32333, L_2 = 17.907, U_2 = 43.11628, L_3 =325.53977 and U_3= 411.9768).

So, the problem can be reformulated using equation (21) "$f_j(x) + \beta(U_j - L_j) \leq (U_j)$" as in P2:

Min / Solution	f_1	f_2	f_3
x^*_1	2.25556	67.82900	281.67200
x^*_2	2.94291	17.90700	411.97680
x^*_3	3.95044	96.68600	150.04640

Table 1. Payoff Table

P2: Max β

s.t.

$720 x_1 + 107 x_2 + 7080 x_3 + 134 x_5 + 1000 x_6 \geq 5000,$

$0.2 x_1 + 10.1 x_2 + 13.2 x_3 + 0.75 x_4 + 0.15 x_5 + 1.2 x_6 \geq 12.5,$

$344 x_1 + 460 x_2 + 1040 x_3 + 75 x_4 + 17.4 x_5 + 240 x_6 \geq 2500,$

$18 x_1 + 151 x_2 + 78 x_3 + 2.5 x_4 + 0.2 x_5 + 4 x_6 \geq 63,$

$x_1 \leq 6.0, \quad x_2 \leq 1.0, \quad x_3 \leq 0.25, \quad x_4 \leq 10.0, \quad x_5 \leq 10.0, \quad x_6 \leq 4.0,$

$0.225 x_1 + 2.2 x_2 + 0.8 x_3 + 0.1 x_4 + 0.05 x_5 + 0.26 x_6 + 0.44067 \beta \leq 3.3233$

$10 x_1 + 20 x_2 + 120 x_3 + 25.20928 \beta \leq 43.11628$

$24 x_1 + 27 x_2 + 15 x_4 + 1.1 x_5 + 52 x_6 + 86.43703 \beta \leq 411.976$

$x_i \geq 0 \ \forall i, \beta \geq 0.$

Solving the model $P2$ by using STORM package the following solution: X = (2.8097, 0, 0, 10, 10, 2.5394), β = 0.59576 and the value of each objective function is as follows. f_1= 2.79243, f_2 = 28.097, and f_3=360.482.

From the above result we can conclude that the solution fits the DM's preferences where the values of the objective functions located between the lower and upper bounds of each objective function that were calculated from equation (17-20) to represent the DM's linguistic preferences of each objective function in the mathematical model.

9. Conclusion

For solving MODM problem, Werners' method [8] considers that the DM has to construct and modify the membership function for each objective. Tapia and Murtagh [11] consider that the DM has selection criteria of the best compromise solution and non-negative underachievement that he/she is willing to accept for each objective function. But, in most practical cases the DM has no knowledge about the mathematical model. However, using the linguistic evaluation approach proposed in this paper, the DM will be able to express his/her preferences in a more direct and adequate way when he/she is unable to express it precisely in the sense of using the natural language in the interaction, a preferred compromise solution is easy to find in the sense of considering the DM's linguistic evaluation of each objective function, and a minimum execution time as a result of a minimum interaction time.

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