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Financial Performance in Manufacturing Firms: A Comparison Between Parametric and Non-Parametric Approaches

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This paper provides a methodological analysis of credit risk in manufacturing firms by using two different credit scoring approaches. The first is the traditional discriminant approach for bankruptcy prediction based on a logistic regression model, whereas the second, data envelopment analysis, is a nonparametric approach for measuring firms' efficiency that does not require ex-ante information on bankrupted firms. By using a manufacturing sample of both healthy and bankrupted firms during the period 2003–09 we provide an in-depth comparison of discriminant analysis and data envelopment analysis and conclude that a correct evaluation of firms' credit worthiness is the result of successive fine-tuning procedures requiring the use of multiple methodological tools.
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The study of a firm's financial performance is relevant in the context of the present economic weakness, as it allows us to understand whether significant threats to economic recovery do exist and whether investment decisions by firms may stimulate and sustain economic growth in the medium to long term.

Firms can be ranked according to their degree of financial constraint, which in turn depends upon macroeconomic factors (such as the cycle or structural characteristics of the economy) and individual characteristics related to the economic and financial position of each firm.

A firm's decision to invest may be affected crucially by its rank; and as rank is significantly determined by financial constraint, an understanding of the distribution of such financial constraints is particularly relevant with respect to new investment. New investment, in turn, is crucial to business success. A firm that is willing to seize growth opportunities by investing may be defined as financially constrained when the amount of internally generated funds is not sufficient to finance investment activity and it cannot access an adequate amount of external resources (debt and/or equity). Indeed, several definitions of credit or financial constraint have been proposed by the relevant literature. Kaplan and Zingales [1997] refer to a wedge between the internal and external cost of funds, whereas Hall [2002] refers to a situation in which there is a wedge between the rate of return required by an entrepreneur investing his own funds and that required by external investors. Thus, there is currently no general agreement on how financially constrained firms can be identified empirically.

The debate concerning the measurement of financial friction at the firm level may gain interesting input from the field of business failure prediction. The main goal here is to predict bankruptcy risk by developing models of financial failure at the firm level before bankruptcy actually happens.

Although business failure has long been debated in both economic and accountancy research, accurate credit risk analysis has become even more important today than it was in the past due to the recent global financial crisis, which has demonstrated how difficult it is to measure and manage business distress.

In this contribution, we analyze credit risk in manufacturing firms during the period 2003–09 by using two alternative approaches: the first is the traditional discriminant analysis approach for bankruptcy prediction, based on a logistic regression model; whereas the second, data envelopment analysis, is a nonparametric

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approach for measuring efficiency. We propose a combination of these two complementary tools in order to develop more accurate predictions of business failure.

1. Measuring Financial Constraints: A Review of the Empirical Literature

Several methods have been proposed in the empirical literature for measuring financial constraints, and the debate is still controversial. Much of the empirical investigation of firms' investment has adopted the analytical framework proposed by Fazzari, Hubbard, and Petersen [1988], arguing that a positive and significant investment-to-cashflow sensitivity signals financial constraint. However, a number of studies have found that differences in cashflow responsiveness between constrained and unconstrained firms is insignificant or that the investment of unconstrained firms is actually more responsive [Gilchrist and Himmelberg 1995; Kaplan and Zingales 1997; Cleary 1999; Allayannis and Mozumdar 2004].

Following this line of research, Almeida, Campello, and Weisbach [2004] suggested that a better measure of financial constraint is given by the sensitivity of cash to cashflow. Using a sample of manufacturing firms between 1971 and 2000, they demonstrated that financially constrained firms have a positive cash-to-cashflow sensitivity, whereas unconstrained firms do not show any systematic pattern. This is explained on the grounds that in contrast with the liquidity irrelevance that characterizes the unconstrained firm, the constrained firm may be forced to save cash today in order to finance future investment opportunities.

In other works, the assessment of the existence of credit constraints is based on qualitative-type information when a firm's subjective perception of its particular financial position is available [Giudici and Paleari 2000; Canepa and Stoneman 2008]. The main problem here is represented by possible misreporting when a credit-demand point of view is considered.

A demonstration of problems associated with self-reported information may be found in the 10th survey on Italian manufacturing firms by Unicredit [2009], according to which—in contrast with the most established empirical evidence—one would conclude that only a small fraction of manufacturing firms (about 4 percent out of a sample of 5,000 firms) faced some type of financial hindrance during the year 2006.

As an alternative, empirical researchers have proposed a sorting approach, which is based on the idea that a firm's financial status may be categorized on the basis of its specific characteristics. Following this approach, Kaplan and Zingales [1997] and Lamont, Polk, and Saa´-Requejo [2001] proposed indices of financial constraint estimated by using ordered logit models. Whited and Wu [2006] developed an alternative index based on generalized method of moment estimations of a standard intertemporal investment augmented model to account for financial frictions. In these models a firm's financial status is a function of various quantitative explanatory variables.

Hadlock and Pierce [2010] exploited an alternative approach based on qualitative information in order to categorize firms. Annual letters to shareholders and management statements from financial filings provided the necessary information for classifying firms in different risk categories. Using this qualitative categorization, order logit models of quantitative information were estimated in order to test the validity of alternative indices of financial constraint proposed by the empirical literature.

In the field of business failure prediction, while traditional methods were essentially based on an expert's evaluation (such as the so-called five Cs credit analysis[1] [Saunders and Allen 2002, pp. 5–9]), since the 1960s a variety of techniques have been proposed in the empirical literature. Discriminant analysis is one such approach. It is essentially based on the idea that a firm's probability of default may be estimated by using a set of key variables. These variables, appropriately combined together, produce a range of quantitative scores that can be used as a classification tool when combined with an appropriate cut-off point. We refer to the seminal work by Altman [1968] and further developments [Deakin 1972 and Altman, Haldeman, and Narayanan 1977], which employ a linear discriminant model based on accounting data of failed and nonfailed firms in order to determine a firm's bankruptcy risk.

Ohlson [1980] proposed a conditional logistic model that has the advantage of overcoming problems associated with the linear discriminant model, namely the assumption of normality and equal covariances for both failed and nonfailed groups. The peculiar feature of the conditional logistic approach is the way a model's precision is tested by considering both classification and future prediction accuracy. Classification accuracy is assessed on the original database—the data set used in order to specify the model. Following this, prediction accuracy is tested by using a new data set, in order to assess how well the model works for future predictions.

In evaluating prediction accuracy there is no way of adjusting the cut-off point for the distribution in order to reduce simultaneously the two types of classification errors, that is the error of classifying a sound firm

[1]The five Cs of credit are: character, capacity, capital, collateral, and conditions.

as unsound (Type I error) and the error of classifying an unsound firm as sound (Type II error). In practice, as there is a trade-off between the two types of error, a pragmatic rule is adopted depending on the specific aim of the classification and, therefore, on the characteristics of the users of such financial information. Indeed, a bank that is evaluating a firm's financial position is probably more interested in minimizing the cost of making a bad investment (Type II error) due to lending funds to a potentially defaulting customer, whereas a shareholder in an innovative firm may be willing to reduce the cost of underinvestment (Type I error) resulting from not taking advantage of an investment opportunity.

One criticism that has been made of traditional approaches is that they are essentially based on accounting ratios, thus omitting the influence of sectoral and macroeconomic conditions. Another criticism concerns the fact that these models are essentially static and inappropriate for predicting a rare event, such as bankruptcy, due to their reliance on data from an arbitrary period before the extreme event occurs.

In more recent years, which have been characterized by a structural increase in bankruptcy worldwide, new approaches have been explored when appropriate longitudinal data are available. Among the most significant contributions, Shumway [2001] proposed a survival analysis approach, which is able to correct for time spent by a firm in the healthy (nonbankruptcy) group and uses time-varying regressors. By using a panel of quoted firms during the period 1962–92 for a total of more than 3,000 firms including 300 firms which went bankrupt, the author estimated a hazard model, based on maximum likelihood estimates of a particular logit model. Among the regressors the model includes not only traditional accounting ratios but also market-driven variables derived from information at the firm level on market capitalization and stock returns. However, the use of these additional explanatory variables is constrained by the availability of such information.

In another study, Linde and Jacobson [2011] studied a firm's probability of default by using a logistic specification with a panel of almost 17 million quarterly observations of Swedish firms during the period 1990–2009. In order to evaluate the effects of macroeconomic conditions, four aggregate variables (output gap, yearly inflation rate, nominal interest rate, real exchange rate) are incorporated into the model, together with a set of financial ratios. The results support the view that although firm-specific variables are important for ranking firms according to their relative risk propensity, macroeconomic conditions do affect the average default level, and thus are important determinants of a firm's risk level.

It is worth noting that discriminant procedures have been criticized on the grounds that they suffer from some of the failings that typically characterize parametric approaches. One of the criticisms concerns possible endogeneity problems affecting financial distress estimations based on accounting information. Endogeneity arises when the financial indices used as explanatory variables are instead the effects of a particular situation of distress.

Another criticism concerns the selection of the appropriate proportion of failed firms in the final sample, given that bankruptcy is a rare event and, thus, difficult to predict. It has been argued that because the link function is symmetric, a logistic regression tends to underestimate bankruptcy probabilities. As a result, more flexible skewed link functions have been indicated as being more suitable for analyzing binary response data [Stukel 1988; Wang and Dipak 2010].

Using a sample of Italian small- and medium-sized enterprises drawn from the AIDA—Bureau van Dijk database over the years 2005–09, Calabrese and Osmetti [2011] propose a generalized extreme value regression for analyzing default probabilities and find that its predictive performance is better than that of the logistic regression predictive model.

In recent years, nonparametric techniques, such as neural networks and decision trees, have been proposed in the empirical literature. These techniques are based on the machine learning approach, the design and development of algorithms that allow computers to predict behavior based on empirical data. Although neural networks have been widely used for failure prediction, no clear demonstration of their superiority has been provided so far. The major criticism of this methodology is that it is a black-box approach, as it is not possible to check for the internal structure of the networks or have information on the relative importance of the variables used. An application to Italian data developed by Altman, Marco, and Varetto [1994] and based on a sample of 1,000 industrial firms demonstrated that neural networks do not outperform traditional discriminant analysis in their ability to classify sound and unsound firms correctly.

An alternative nonparametric approach to credit scoring is based on the data envelopment analysis methodology [Charnes, Cooper, and Rhodes, 1978], which has the advantage of not depending on the availability of ex-ante information on bankruptcy events. The data envelopment analysis scoring approach is essentially a mathematical programming method to evaluate the relative efficiency of "decision-making units" (DMUs). By converting multiple inputs into multiple outputs, data envelopment analysis computes the relative efficiency scores of each DMU (that is a firm or a bank). This approach has been widely applied in different frameworks; examples of applications to banking and finance are given by Yeh [1996], Troutt, Rey, and Zhang [1996], Simak [1999], Cielen and Vanhoof [1999] and, more recently, Min and Lee [2008].

In the present work we adopt a credit scoring procedure. Our main interest is to provide different methods that may be used as complementary approaches for predicting firms' economic and financial performance. Thus, we first perform a discriminant analysis based on a sample of both failed and nonfailed firms in order to derive an empirical measure of financial worthiness and, implicitly, financial constraint. Default probabilities are estimated by using a logistic model that includes both firm-specific characteristics and financial indices. We then apply the data envelopment analysis approach to the same database used for the logistic discriminant in 2003. By using an appropriate set of financial ratios, firms' credit worthiness is estimated by exploring the relative efficiency of the complete set of firms (both failed and nonfailed). Both methodologies are then applied to the sample of firms in 2009 in order to define appropriate credit scoring suitable for comparison.

2. The Data

Accounting information

Our main sample of firms is derived from the 10th Unicredit survey of Italian manufacturing firms [2009]. This sample is composed of more than 5,000 firms that are representative of the manufacturing sector and extracted from the AIDA database. A rich set of information is collected by this survey, including firm-specific characteristics and investment and innovative activities. This starting sample has been supplemented with a rich set of accounting data. The economic and financial information derived from firms' balance sheets has allowed us to derive the financial indices used in the credit scoring procedures that will be described in the following sections.

Bankruptcy data

Bankruptcy data have been collected from the AIDA database. We extracted a sample of 150 firms that went bankrupt during the years 2005 and 2006. Balance sheet information refers to years 2003 and 2004 in order to have an adequate time span difference (not less than one year) between the last relevant balance sheet and the bankruptcy date.

The sample size was fixed by taking into account two important conditions in order to derive reliable default predictions. Firstly, although firm default is a rare event,[2] it is important to supplement the sample of nondefaulting firms with an adequate number of defaulting firms in order to derive a reliable discriminant rule for predicting a *rare* event. In addition, firms close to bankruptcy may present abnormal accounting data that should be removed, given that discriminant techniques are particularly sensitive to outliers, thus determining a further reduction in bankruptcy observations.

Secondly, the probability of default for the firms on the Italian business register is significantly affected by specific characteristics, such as age, size, and localization. In general, smaller and younger firms localized in southern regions show a higher probability to default compared with older and larger firms localized in northern regions.

In order to take these differences into account we decided to stratify the sample so as to increase the representativeness of our set of bankrupt firms.[3] Stratification was first determined by area, then by age, and then by size.[4] Firms were selected randomly in each stratum, whereas the allocation of the sample across strata was assessed on the basis of a system of weights that were applied to the default probability observed in the Italian Business Register (the reference population) [Cerved 2011].

3. The Discriminant Approach

The logistic discriminant model

We estimate the default probability of a firm by using a logistic discriminant function defined as follows:

$$\log\left(\frac{p}{1-p}\right) = b_0 + b_1x_1 + b_2x_2 + \cdots + b_{k1}x_{k1}, \quad (1)$$

[2]Estimations by the Cerved Group [2007] show that Italy, with 18 cases per 10,000 firms in 2006, has an insolvency ratio that is far below the European average (63 cases). However, it is worth noting that international comparisons should be interpreted with caution, due to different insolvency regimes and to differences in firms' structural characteristics across Europe. In fact, in countries such as Spain, Greece, and Italy, where the proportion of small businesses is higher, the low levels of the insolvency ratio may reflect that insolvent firms opt for voluntary abandonment rather than formal insolvency proceedings.

[3]A table showing the allocation of the sample according to the stratification criteria by area, age, and firm size is available on request.

[4]Cerved Group [February 2011—Rapporti Flash] has estimated that one of the most relevant determinants of the default probability is a firm's location, followed by age and firm size, whereas the sector of activity is not among the most relevant factors.

where

$$p = Prob(D = 1 \mid X). \quad (2)$$

D is our binary dependent variable, which assumes the value of 1 if we observe a default event between years 2005 and 2006 and 0 otherwise; and *X* is the vector of covariates, that is firm-specific characteristics and financial indices that are observed in years 2003 and 2004.

We have included a set of variables that are commonly considered good predictors of the outcome event in the relevant literature:[5]

- a measure of a firm's leverage (LEV), the ratio of total debts to net capital, which is expected to affect the default probability positively, as a highly leveraged structure may worsen the perceived financial risk;
- a measure of short-term indebtedness (CL_S), the ratio of current liabilities to sales, whose expected sign is positive, given that a firm with a high short-term debt may find it difficult to borrow additional resources to finance its short-run activities and may thus be close to insolvency;
- the "acid-test ratio" (ACID); this measures the extent to which short-term debt is covered by short-term liquidity. It is the sum of cash, accounts receivable, and short-term investments relative to current liabilities. Creditors prefer a high ACID ratio as it reduces their risk. We thus expect a negative sign;
- firm operating profitability (ROS), proxied by the ratio of operating margins to sales. We expect a negative effect on the default risk, as the higher a firm's profitability the higher the flow of internal resources available to cover debt exposure should be;
- the firm's interest burden, proxied by the interest payment to sales (IR) ratio, which is expected to positively affect the default probability, given that a high interest burden may worsen the financial risk associated with external finance. We have used a dummy variable assuming the value of 1 when a firm shows an interest burden ratio higher than 5 percent, which identifies the last 5 percent of the IR distribution, and 0 otherwise, in order to capture the effect of those firms that are potentially financially constrained;
- structural characteristics, captured by variables AGE (years in business) and SIZE, proxied by a firm's total assets (logarithmic values). We expect a negative effect of both these variables, as agency costs related to indebtedness are expected to be higher for those firms with a low reputation or contractual power, such as those which are smaller or less well established.

Table 1. Default Probability—Logistic Discriminant

	Coefficient Sign	*p*-value (year 2003)	*p*-value (year 2004)
constant	+	0.0009	0.1085
ACID	−	0.0172	0.0149
LEV	+	0.0002	0.0423
CL_S	+	0.1473	0.0607
ROS	−	0.0009	<0.0001
dIR	+	0.0085	0.6346
L_TA	−	<0.0001	<0.0001
AGE	−	0.0004	0.0151
dNW	−	<0.0001	0.0002
dNE	−	0.0002	0.0657
dC	−	0.113	0.2481

year 2003: N = 4,100
Percent Concordant: 93.5 percent
LR chi^2 (10) 284.2
year 2004: N = 4,607
Percent Concordant: 90.8 percent
LR chi^2 (10) 201.9

Estimation results are shown in Table 1. We estimate default probabilities within one and two years. In the first case the model is computed by using predictors observed in the year 2004, whereas in the second case we use information for the year 2003. In both models our variables present the expected signs, although it is worth noting that the statistical results are better when information two years before bankruptcy is used.[6] Thus, by using accounting information from two years prior the default event, we can build what we expect to be a more accurate prediction model.

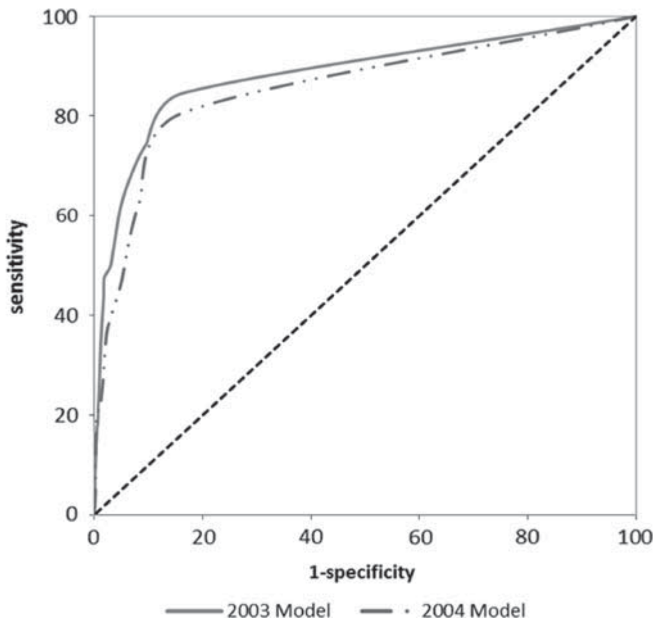
Classification accuracy

Classification accuracy is evaluated by using the samples of firms used to predict default probabilities in the years 2003 and 2004. Different cut-off points are associated with a tradeoff between Type I Error (False Positives) and Type II Error (False Negatives). If a cut-off point of 0.5 is selected from the 2003 model, only 5 firms out of 78 are correctly classified as bankrupt.

[5]Descriptive statistics are reported in the Appendix.

[6]The results show that the 2003 regression performed better in terms of overall significance, as confirmed by the percentage concordant index and by the χ^2 test based on the log-likelihood of the regressions.

Figure 1. ROC Curve for Logistic Discriminant Model



Classification table

Cutoff point	2003 Model: default firms correctly classified	sound firms wrongly classified	Type I Error	Type II Error	2004 Model: default firms correctly classified	sound firms wrongly classified	Type I Error	Type II Error
							98.5	.
0	78	4022	98.1	.	67	4540	93.2	0.3
0.02	66	660	90.9	0.4	54	738	90	0.6
0.04	58	385	86.9	0.5	42	379	87.8	0.8
0.06	53	270	83.6	0.7	31	223	84.4	0.9
0.08	48	192	80	0.8	27	146	80.8	1
0.1	42	144	77.4	0.9	24	101	80.2	1.1
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.

(Area under the curve: 0.951(2003 Model) and 0.9083 (2004 Model))

Using 2004, the same cut-off produces an even worse prediction (1 out of 67).

The Receiver Operating Characteristic (ROC) curve is then used as a diagnostic test for accuracy. It plots the true positive rate (sensitivity) against the false positive rate (1-specificity) for different cut-off points, as shown in Figure 1. The area under the curve describes how well the classification rule works: an ROC curve which passes through the upper left-hand corner would indicate an optimal discrimination (100 percent sensitivity and 100 percent specificity), whereas the 45° degree line indicates a situation of irrelevance, as for each cut-off point one would observe the same error for both types (no trade-off). Thus, the closer the ROC curve is to the upper left-hand corner, the higher the accuracy of the discrimination rule.

By overlapping the ROC curves for years 2003 and 2004 it is possible to show better performance of the 2003 model. For each cut-off point, classification based on the 2003 model yields Type I and Type II Errors that are lower than for the 2004 model.

If a cut-off point of 0.02 is fixed, a Type II Error of 0.5 is obtained with the 2003 model (66 out of 78 bankruptcy cases correctly predicted). However, as at this cut-off point we also wrongly classify as unsound 660 out of 4,022 healthy firms, we prefer to accept a small increase in Type I Error in order to reach a better classification for the group of healthy firms. Thus, a cut-off point of 0.04 seems to be a reasonable compromise (58 out of 78 bankruptcy cases correctly predicted and 3,637 out of 4,022 sound firms correctly classified) to be used for future prediction.

The set of estimated coefficients from the logistic discriminant together with the adjusted cut-off point will be used to predict business failure. We perform a new logistic discriminant based on the previously saved set of rules and on a new data set. The new data

Table 2. Frequency Distribution of Firms' Default Probability

Total No. of Firms: 3,424		No. of Firms Above the Cut-Off Point: 200		No. of Firms Below the Cut-Off Point: 3,224	
Quantile	Prob. estimate	Quantile	Prob. estimate	Quantile	Prob. estimate
100%Max	0.856470587	100% Max	0.8564706	100% Max	0.039837629
99%	0.214908159	99%	0.7020042	99%	0.034542941
95%	0.047518534	95%	0.39709	95%	0.021215554
90%	0.02223708	90%	0.3027572	90%	0.012868728
75% Q3	0.005199702	75% Q3	0.1405008	75% Q3	0.003673321
50% Median	0.000746195	50% Median	0.0780178	50% Median	0.000596812
25% Q1	0.000075912	25% Q1	0.054063	25% Q1	0.000063037
10%	0.000005003	10%	0.0465096	10%	0.000004221
5%	0.000000628	5%	0.0430681	5%	0.000000526
1%	0	1%	0.0407516	1%	0
0%Min	0	0%Min	0.0404833	0%Min	0

set comprises the Unicredit sample of firms observed in 2009 and the same accounting variables as for 2003.

Prediction

To predict business failure, we decided to divide firms into four risk classes according to estimated probability intervals and relative frequency distributions. By considering the subsample of Unicredit firms operating in 2009 (3,424 firms), only 200 firms presented an estimated default probability greater than the fixed cut-off point, as shown in Table 2. By splitting the 2009 sample into two subsamples, the sample with default probabilities higher than 0.04 was further divided into two additional subsamples: the first group, representing the last 90th percentile, can be regarded as the group of *risky* firms (20 firms in 2009), the rest of the distribution (179 firms) can be regarded as *critical* firms. The other subsample with default probabilities lower than 0.04 was divided into *good* firms (the last 75th percentile corresponding to 2,419 firms) and *excellent* firms (the first 25th percentile corresponding to 806 firms).

We also applied the same classification to the 2003 sample of firms in order to derive a cross tabulation with frequency distributions across the four risk classes at time T and T+6. This representation allows us to investigate persistence patterns and transition probabilities across risk classes during a six-year time span.

Table 3. Discriminant Analysis Association Between Risk Classes—Years 2003 and 2009

2003	2009				
	Risky	Critical	Normal	Excellent	Total
Risky	3	5	4	0	12
	0.1	0.17	0.14	0	0.41
	25.0	41.67	33.33	0	—
	23.08	4.39	0.19	0	—
Critical	7	69	133	0	209
	0.24	2.37	4.57	0	7.19
	3.35	**33.01**	63.64	0	—
	53.85	60.53	6.38	0	—
Normal	3	40	1,840	128	2,011
	0.1	1.38	63.27	4.4	69.15
	0.15	1.99	**91.5**	6.36	—
	23.08	35.09	88.25	18.39	—
Excellent	0	0	108	568	676
	0	0	3.71	19.53	23.25
	0	0	15.98	**84.02**	—
	0	0	5.18	81.61	—
Total	13	114	2,085	696	2,908
	0.45	3.92	71.7	23.93	100

Note: In each cell: frequency, percent, row percent, column percent. Values in bold: persistence rate.

Table 3 shows high degrees of persistence in the normal and excellent classes: 91 percent and 84 percent of firms which had been classified, respectively, as normal and excellent in 2003 were still in the same category in 2009. Persistence in the critical group of firms is much lower but not negligible: 30 percent of firms that were classified as critical in 2003 were still in the same situation (69 out of 209 firms) in 2009.

Finally, only 12 firms in the critical group in 2003 were still present in 2009. Eight of them were still in an unsafe condition (critical or risky), but the most interesting consideration here is that Table 4 shows that

Table 4. Firms by Risk Class—Exit Rates

Class_discr	Firms Observed in 2003	Firms Observed in 2003 and in 2009	Exit Rate (%)
Risky	45	12	73
Critical	398	209	47
Normal	2,742	2,011	27
Excellent	915	676	26
Total	4,100	2,908	29

73 percent of the risky firms in 2003 had ceased business before 2009. Although the exit rate may be affected by factors other than business management (such as data availability in both years and/or mergers and acquisitions), it is worth noting that the exit rate is much lower in the other classes—26 percent in both the excellent and normal groups of firms and 47.5 percent in the critical group.

4. An Alternative Approach to Credit Scoring: Data Envelopment Analysis

In the previous sections we have described the use of discriminant analysis to develop adequate credit scoring indices. We have underlined how the results crucially depend on the availability of a sufficiently large amount of information on bankrupted firms. Typically, such an approach implies that the number of bankrupted firms is relatively small compared with the overall number of firms under investigation. This fact may crucially affect the results of discriminant analysis, which may underestimate default probabilities.

Our task is therefore to develop a methodology that can be used as a complement to discriminant analysis to help assess the financial and economic position of a firm more accurately. We therefore apply data envelopment analysis in order to rank firms according to a financial score derived from a nonparametric methodology. Data envelopment analysis has been used widely to analyze the efficiency and productivity of firms in the economy since the seminal contribution by Charnes, Cooper, and Rhodes [1978], with applications to many different sectors, contexts, and activities. As a nonparametric approach, it can easily be applied to different frameworks, particularly when comparisons between DMUs are fundamental either for policy analysis or for other economic choices.

Charnes, Cooper, and Rhodes (CCR) [1978] proposed the basic data envelopment analysis model (hereafter the CCR model), which has since been extended to a variety of different hypotheses. The CCR model implies that there are n DMUs that convert the same m inputs into the same s outputs. In general terms, the jth DMU uses an m-dimensional input vector $\mathbf{x}$ to produce an s-dimensional output vector $\mathbf{y}$.

This implies the following maximization problem:

$$Max\,\theta_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \qquad (3)$$

subject to:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \qquad u_r \ge 0, \qquad v_i \ge 0 \;\; \forall \;\; r; i; j, \qquad (4)$$

where subscript o indicates the particular DMU_j being evaluated, and u_r ($r = 1, 2\ldots, s$) and v_i ($v = 1,2 \ldots, m$) are respectively output and input weights, which are required to be nonnegative.

The previous definition of the data envelopment analysis problem relies on the concept of input and output variables that may be minimized or maximized. In particular, one can think of inputs as being minimized while satisfying at least the given output levels, or outputs being maximized without requiring more of any of the given inputs. The first approach refers to the so-called *input-oriented* model, whereas the second refers to the *output-oriented* model.

In our analysis we have adopted the *input-oriented* model, which can in our opinion be better applied to a financial problem in that a firm is trying to minimize financial expenses for a given output variable.[7] One should note that the CCR version of the data envelopment analysis model implies constant returns to scale: in other words, one can think of the existence of a linear and significant relationship between input and output variables. We therefore tested for the existence of a significant linear relationship with respect to the input and output variables before deciding to apply this model. The application of the data envelopment analysis methodology to credit scoring is relatively recent, thus suggesting that this is an applied field of research which has not yet been exploited.

We decided to use some of the financial and economic ratios which were selected for the logistic discriminant model described in the previous section (*ACID, LEV, ROS, IR*). In addition, we decided to include the Net Capital to Total Assets Ratio (*CN_A*) and the Short-Term Debt to Total Assets ratio (*CL_A*), which may better represent the data envelopment

[7]This problem setting is also used by Min and Lee [2008].

Table 5. Frequency Distributions of Data Envelopment Analysis Scores in 2003 and 2009

Quantile	Scores Estimate	
	Year 2003 No. of firms: 4,166	Year 2009 No. of firms: 3,845
100% Max	1.000	1.000
99%	0.975	1.000
95%	0.666	0.582
90%	0.547	0.443
75% Q3	0.364	0.286
50% Median	0.228	0.172
25% Q1	0.143	0.091
10%	0.094	0.044
5%	0.069	0.022
1%	0.029	0.000
0% Min	0.011	0.000

Table 6. Firms by Risk Class

Class_DEA	Firms Observed in 2003	%	Firms Observed in 2009	%
Risky	208	5.0	192	5.0
Critical	833	20.0	769	20.0
Normal	2,083	50.0	1,922	50.0
Excellent	1,042	25.0	962	25.0
Total	4,166	100.0	3,845	100.0

Table 7. Data Envelopment Analysis Association Between Risk Classes—Years 2003 and 2009

2003	2009				
	Risky	Critical	Normal	Excellent	Total
Risky	18	68	40	6	132
	0.56	2.13	1.26	0.19	4.14
	13.6	51.52	30.3	4.55	—
	14.17	10.76	2.42	0.78	—
Critical	40	250	282	28	600
	1.26	7.84	8.85	0.88	18.83
	6.67	**41.67**	47	4.67	—
	31.5	39.56	17.03	3.63	—
Normal	64	277	1,003	270	1,614
	2.01	8.69	31.47	8.47	50.64
	3.97	17.16	**62.14**	16.73	—
	50.39	43.83	60.57	34.97	—
Excellent	5	37	331	468	841
	0.16	1.16	10.39	14.68	26.39
	0.59	4.4	39.36	**55.65**	—
	3.94	5.85	19.99	60.62	—
Total	127	632	1,656	772	3187
	3.98	19.83	51.96	24.22	100

Note: In each cell: frequency, percent, row percent, column percent. Values in bold: persistence rate.

analysis problem set. The inputs to be minimized are *ACID*, *LEV*, *CL_A*, and *IR*, while the operating profitability ratio (*ROS*) and the Net Capital to Total Assets Ratio (*CN_A*) are set as output variables.

The solution of the optimization problem described in equation (3) determines n scores that may be thought of as financial stability scores. The distribution of data envelopment analysis scores in 2003 and 2009 is shown in Table 5. In order to classify firms according to their score, we analyzed the distribution of the scores; and we propose a classification that defines the top 25 percent of the distribution as excellent and the bottom 5 percent as risky. Having defined the extreme scores, one should attempt to define the intermediate quintiles. The result is a classification which implies that almost 5 percent are risky, 20 percent are critical, 50 percent are good, and 25 percent excellent. The analysis of the data confirms the soundness of such a classification: the ex-post evaluation of the scores attached to the subsample of bankrupted firms used for the discriminant analysis confirms that all firms have been correctly classified as risky.

This distribution is stable, as it holds in both years (Table 6). However, although the distribution is stable, we can observe movements within these financial states during the time span considered. We can analyze such movements by looking at Table 7, which shows the flows of firms from one state to another between the two periods. Thus, more than 55 percent remain excellent, whereas more than 62 percent remain normal, and almost 42 percent and almost 14 percent stay critical and risky, respectively.

On the whole almost 22 percent of firms show an upward shift in the ranking, whereas almost 24 percent show a downward shift, thus implying a downgrade of their financial condition. These ratios are derived by dividing the sum of the lower (upper) off-diagonal values of the matrix represented in Table 7 by the total number of firms. In particular, 51.4 percent of firms that were either critical or risky remained so, whereas 84.4 percent of those that were normal or excellent remained so, thus suggesting that persistence does characterize firms' financial condition. We will return to this point in the following section.

Table 8. Association between Discriminant Analysis and Data Envelopment Analysis Approaches: 2003

Data Envelopment Analysis	Discriminant Analysis				
	Risky	Critical	Normal	Excellent	Total
Risky	15	63	116	13	207
	0.37	1.54	2.83	0.32	5.05
	7.3	30.43	56.04	6.28	—
	33.33	15.83	4.23	1.42	—
Critical	14	122	583	103	822
	0.34	2.98	14.22	2.51	20.05
	1.7	**14.84**	70.92	12.53	—
	31.11	30.65	21.26	11.26	—
Normal	14	172	1,467	394	2,047
	0.34	4.2	35.78	9.61	49.93
	0.68	8.4	**71.67**	19.25	—
	31.11	43.22	53.5	43.06	—
Excellent	2	41	576	405	1,024
	0.05	1	14.05	9.88	24.98
	0.2	4	56.25	**39.55**	—
	4.44	10.3	21.01	44.26	—
Total	45	398	2,742	915	4100
	1.1	9.71	66.88	22.32	100

Measures of Associations

Statistic	DF	Value	Prob
Chi-square (a)	9	470.6916	<0.0001
Spearman Correlation Coefficient (b)	—	–0.35655	<0.0001

Note: In each cell: frequency, percent, row percent, column percent. Values in bold: persistence rate.
(a) Association between risk classes (4 classes).
(b) Measure of association based on the ranks of the firms' scores. Prob > |r| under H0: Rho = 0. The correlation is negative as firms are inversely ranked: DEA scores imply that a value close to one is related to a good financial condition, whereas the DA scoring rule implies that such a condition is realized when the score tends to zero.

Scoring performance: A comparison between discriminant analysis and data envelopment analysis approaches

These results underline a significant difference between the two proposed scoring methodologies, in that discriminant analysis is a parametric procedure, whose outcomes crucially depend on the choice of the Type I and Type II Error classification one is willing to accept. In our sample of firms, we decided to choose a probability threshold (0.04) which enabled us to identify more than 74 percent of bankrupted firms correctly. On the other hand, data envelopment analysis is a nonparametric methodology, which implies an optimization problem. Thus, it does not depend on an a priori hypothesis concerning the model being estimated or simulated.

Tables 8 and 9 permit a more in-depth examination of discriminant analysis and data envelopment analysis results by presenting cross classifications that enable us to verify and test for the degree of association of the two methodologies. If we consider the values on the main diagonal of the 4×4 matrices that compare discriminant analysis and data envelopment analysis classifications, we note that 49 percent of firms in 2003 and more than 51 percent are accordingly classified by both methodologies. If one considers a less restrictive classification, say, good firm (excellent or normal) and bad firm (critical or risky), these percentages significantly increase to 74.5 and 77.8 percent respectively in 2003 and 2009. This evidence is then reflected in the chi-square tests on the degree of association of the two classifications and the Spearman Correlation Coefficient.

Table 9. Association between Discriminant Analysis and Data Envelopment Analysis Approaches: 2009

Data Envelopment Analysis	Discriminant Analysis				
	Risky	Critical	Normal	Excellent	Total
Risky	5	11	35	2	53
	0.15	0.32	1.02	0.06	1.55
	9.4	20.75	66.04	3.77	—
	25	6.15	1.45	0.25	—
Critical	9	85	556	74	724
	0.26	2.49	16.27	2.17	21.19
	1.24	**11.74**	76.8	10.22	—
	45	47.49	23.02	9.22	—
Normal	5	73	1,337	393	1,808
	0.15	2.14	39.13	11.5	52.91
	0.28	4.04	**73.95**	21.74	—
	25	40.78	55.36	48.94	—
Excellent	1	10	487	334	832
	0.03	0.29	14.25	9.77	24.35
	0.12	1.2	58.53	**40.14**	—
	5	5.59	20.17	41.59	—
Total	20	179	2,415	803	3,417
	0.59	5.24	70.68	23.5	100

Measures of Associations

Statistic	DF	Value	Prob
Chi-square (a)	9	383.5786	<0.0001
Spearman Correlation Coefficient (b)	—	−0.40018	<0.0001

Note: In each cell: frequency, percent, row percent, column percent. Values in bold: persistence rate.
(a) Association between risk classes (4 classes).
(b) Measure of association based on the ranks of the firms' scores. Prob > |r| under H0: Rho = 0. The correlation is negative as firms are inversely ranked: DEA scores imply that a value close to one is related to a good financial condition, whereas the DA scoring rule implies that such a condition is realized when the score tends to zero.

This comparison between discriminant analysis and data envelopment analysis scores highlights the different methodological foundations of the two approaches, and suggests that they can be used as complements in the analysis of firms' financial worthiness.

In addition, another interesting difference becomes apparent if one considers class-movements between the reference years recorded according to the two different approaches, shown in Table 10. Firm performance seems to be more conservative according to the discriminant analysis approach: 85.3 percent of the firms do not change risk class during the observed period, and only 9.3 and 5.4 percent of the sample experiment, respectively, experience an upgrading or a downgrading. Conversely, firms classified according to the data envelopment analysis approach show a higher sensitivity to movements between classes. As we showed in the previous section, firms remaining in the same class represent 54.6 percent of the sample, while downward and upward movements involve, respectively, 23.6 and 21.8 percent of the sample.

Table 10. Movements Between Risk Classes

Status	Discriminant Analysis Classification		Data Envelopment Analysis Classification	
2009 with respect to 2003	Frequency	Percent	Frequency	Percent
Stable	2,480	85.28	1,739	54.6
Downgrading	158	5.43	754	23.7
Upgrading	270	9.28	694	21.8
Total	2,908	100.0	3,187	100.0

In addition, our approach contrasts with previous validations and comparisons of the two methodologies [Min and Lee 2008]. First, we do not use regression analysis to validate data envelopment analysis scores either by simply regressing such scores with respect to the input and output variables used in the data envelopment analysis optimization procedure. Second, we do not apply a logit (probit) regression to a dichotomous variable, derived from the application of a given cut-off point (such as the median value) to the distribution of data envelopment analysis scores, that is dependent on the same explanatory variables used in the linear regression.

Indeed, an approach such as that of Min and Lee is self-reinforcing and self-validating as data envelopment analysis scores are derived from an optimization process that uses the same variables as those used in the regression analysis. On the contrary, our validation approach compares the raw outcomes of the two procedures and thus enables us to state clearly the advantages and disadvantages of the two methodologies.

5. Conclusions

Firms' financial performance is crucial as it determines future decisions and actions which, in turn, affect growth at the micro (company) and macro (economy-wide) levels. In particular, we have emphasized and reviewed how firms' financial performance may affect their investment decisions. Thus, it is crucial to be able to determine and classify a firm's financial worthiness.

We have therefore analyzed the performance of a representative sample of Italian manufacturing firms, by applying a parametric approach (logistic discriminant) and a nonparametric approach (data envelopment analysis). The comparison between the different approaches is necessary, as the evaluation of a firm's financial performance is the result of fine-tuning procedures that require the use of multiple methodological tools.

Discriminant analysis is based on the assumption of a given distribution of a firm's default probability, which is assumed to be logistic. Such a procedure enables one to estimate and then forecast a firm's default probability. However, we have emphasized that one significant drawback lies in the fact that in order to estimate such probabilities one needs to gather information on firms which are already bankrupt. Typically, the number of these latter firms is relatively small compared with that of nonbankrupt firms; this fact produces a bias in that the estimated default probabilities are underestimated.

Thus, we have proposed a methodology that does not require ex-ante information on bankrupt firms. Data envelopment analysis is a nonparametric approach that enables one to rank firms according to their efficiency or other measures of financial worthiness by applying appropriate linear programming models. In particular, we have chosen the CCR input-oriented version of data envelopment analysis, which implies the minimization of given input variables for given outputs. This choice is based on some experiments that enabled us to verify that the relationship between the inputs and outputs we have chosen is linear and, therefore, the constant-returns-to-scale hypothesis implied by the CCR model is not ruled out. In addition, the input-oriented model seems better adapted to the setting of firms' financial and economic problems.

All in all, the data envelopment analysis model performs better than the discriminant analysis approach, in that it enables us to classify as failed those firms that indeed went bankrupt. In addition, such a modeling approach does not depend on data availability on bankrupt firms, a fact that crucially affects the discriminant analysis results . Our results enable us to achieve a more comprehensive picture of firms' financial performance, and we are able to predict defaults of those bankrupted firms whose balance sheets were used in the discriminant analysis. The discriminant analysis, on the contrary, correctly predicts default probability in 75 percent of cases.

The analysis presented in this study therefore represents a fundamental and necessary background for investigating the aforementioned relationship between financial worthiness and investment and firms' performance, in particular, with respect to investment and innovation decisions. In addition, our analysis—which employs financial and economic indicators parsimoniously—can be adopted when previous information on bankruptcy is unavailable or insufficient, and it may find specific application with reference to small- and medium-sized businesses for which financial and economic data may be lacking to a greater or lesser extent.

APPENDIX

Table A1. Descriptive Statistics by Risk Class *(DEA—year 2009)*

Variable	Description	No. Firms		Mean	Median	Min	Max
ACID	*Acid test—Liquid Assets/Current Liabilities*	*3,845*	*all*	1.20	0.94	0.20	5.75
		192	*risky*	0.77	0.68	0.20	3.60
		961	*critical*	0.88	0.79	0.27	4.46
		2,883	*normal*	1.07	0.92	0.21	5.22
		962	*excellent*	1.80	1.62	0.24	5.75
LEV	*Leverage—Total Debts/Net Capital*	*3,845*	*all*	4.25	1.97	−12.70	73.06
		192	*risky*	17.76	22.59	−12.70	73.06
		769	*critical*	10.91	8.13	0.54	60.89
		1,922	*normal*	2.51	2.04	0.15	47.51
		962	*excellent*	0.72	0.60	0.05	4.57
CN_A	*Net Capital/Total Assets*	*3,845*	*all*	0.32	0.30	−2.04	0.92
		192	*risky*	−0.10	0.02	−2.04	0.05
		769	*critical*	0.11	0.10	0.01	0.61
		1,922	*normal*	0.32	0.30	0.02	0.87
		962	*excellent*	0.57	0.57	0.17	0.92
CL_S	*Current Liabilities/Sales*	*3,845*	*all*	0.55	0.48	0.12	2.49
		192	*risky*	0.83	0.72	0.14	2.49
		769	*critical*	0.67	0.62	0.17	2.45
		1,922	*normal*	0.56	0.50	0.12	2.46
		962	*excellent*	0.38	0.30	0.12	2.48
CL_A	*Current Liabilities/Total Assets*	*3,845*	*all*	0.48	0.46	0.04	2.70
		192	*risky*	0.78	0.76	0.19	2.70
		769	*critical*	0.65	0.68	0.07	0.96
		1,922	*normal*	0.47	0.47	0.08	0.95
		962	*excellent*	0.31	0.30	0.04	0.77
ROS	*Return on Sales*	*3,845*	*all*	0.02	0.03	−0.46	0.28
		192	*risky*	−0.12	−0.10	−0.46	0.04
		769	*critical*	0.00	0.02	−0.45	0.15
		1,922	*normal*	0.02	0.03	−0.43	0.25
		962	*excellent*	0.05	0.04	−0.40	0.28
IR	*Interest Payment/Sales*	*3,845*	*all*	0.018	0.013	0.000	0.398
		192	*risky*	0.029	0.022	0.000	0.104
		769	*critical*	0.026	0.021	0.000	0.147
		1,922	*normal*	0.020	0.014	0.000	0.209
		962	*excellent*	0.007	0.002	0.000	0.398
TA	*Total Assets (euros)*	*3,845*	*all*	17,436,284	6,293,267	736,368	321,386,959
		192	*risky*	15,643,710	3,910,625	813,616	303,830,920
		769	*critical*	11,918,528	4,441,134	736,368	261,845,800
		1,922	*normal*	19,322,470	7,218,616	740,810	320,259,996
		962	*excellent*	18,293,980	7,073,901	793,142	321,386,959
AGE	*Firm Age in Years*	*3,845*	*all*	31	28	2	259
		192	*risky*	29	22	5	259
		769	*critical*	26	23	2	259
		1,922	*normal*	32	29	2	259
		962	*excellent*	34	30	2	159

Note: Observations below the 1th or above the 99th percentile excluded. Significance of italic: Number of firms.

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