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A Wald Test for Spatial Nonstationarity

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ABSTRACT

A test strategy consisting of a two-step Lagrange multiplier test was recently suggested as a device to reveal spatial nonstationarity, spurious spatial regression and presence of a spatial cointegrating relationship between two variables. Due to the well known radicality of such pre-tests in finite samples, the present paper suggests a Wald post-test, based on maximum likelihood estimation. The finite-sample distribution of the test under nonstationarity is derived using Monte Carlo simulation and applied to an empirical example.

Keywords: Spatial nonstationarity, spurious regression, Wald tests, Lagrange multiplier tests, Regional Economics.

Un contraste de Wald de No estacionariedad espacial

RESUMEN

Se ha propuesto recientemente una estrategia de contraste basada en el Multiplicador de Lagrange en dos etapas para analizar no estacionariedad espacial, regresión espacial espurea y la presencia de relaciones de cointegración en el caso bivariante. Como es conocido, estos métodos condicionados tienen problemas en muestras finitas por lo que en el trabajo se presenta un contraste de Wald, basado en la estimación de máxima verosimilitud. En el trabajo se obtiene la distribución en muestra finita del contraste bajo la hipótesis de no estacionariedad mediante simulaciones de Monte Carlo, y se aplica a un ejemplo concreto. La distribución obtenida para el contraste de Wald parece tener unas colas más densas que la distribución tradicional, chi-cuadrado con un grado de libertad.

Palabras clave: No estacionaridad espacial, Regresión espurea, Contraste de Wald, Contraste de los Multiplicadores de Lagrange, Economía espacial.

Clasificación JEL: C21, C40, C51, J60, R10.

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1. INTRODUCCIÓN

Spatial regression has been discussed widely in books dedicated to developments in spatial econometrics, notably by Anselin (1988a), and Anselin and Florax (1995). The consequenses for estimation and inference in the presence of stable spatial processes have been extensively investigated (Haining 1990; Anselin 1988a; Bivand 1980; Richardson 1990; Richardson and Hèmon 1981; Clifford and Richardson 1985; Clifford, Richardson and Hèmon 1989). A recent study (Fingleton 1999) takes the first steps into analyses of implications of spatial unit roots, spatial cointegration and spatial error correction models. A follow-up to this study is found in Mur and Trivez (2003), where the concept of spurious spatial regression is established in a framework of spatial trend (non)stationarity. In Lauridsen (2004) estimation of spatial error-correction models using an IV approach is investigated. Further, Lauridsen and Kosfeld (2004) and Kosfeld and Lauridsen (2004) establish and apply a two-step Lagrange Multiplier test for nonstationarity.

The topics studied in the present investigation may be viewed as generalisations of common topics studied in a basin of time series literature. For example, two survey papers on the subject of unit roots in economic time series data, Diebold and Nerlove (1990) and Campbell and Perron (1991) cite over 200 basic sources on the subject. The literature on unit roots and cointegration is one of the most rapidly moving target in econometrics. Stock’s (1994) survey adds hundreds of references to those in the aforementioned surveys and brings the literature up to date as of then. Useful basic references on the subjects are Box et al. (1994); Judge et al. (1985); Mills (1990); Granger and Watson (1996); Granger and Newbold (1996); Hendry et al. (1984); Geweke (1984); Harvey (1989, 1990); Enders (1995); Hamilton (1994); and Patterson (2000).

The present paper refines recent suggestions. Specifically, Fingleton (1999) suggests that “very high” values of the Moran test for spatial residual autocorrelation indicate spatial nonstationarity and spurious regression. It is, however, left as an open question how to distinguish between stationary positive autocorrelation and nonstationarity. Lauridsen and Kosfeld (2004) shows that a two-step Lagrange multiplier (LM) test for positive residual autocorrelation can provide a better founded basis to separate these two cases and that the same procedure works as a diagnostic for spurious regression and spatial cointegration. The practical applicability of the suggested LM test approach was illustrated in Lauridsen and Kosfeld (2004) and Kosfeld and Lauridsen (2004), using cases from recent empirical research. But they did not treat the well known radicality problem of the LM test, due to its high finite-sample power function. It is well known that the LM test, the Likelihood Ratio (LR) test and the Wald test for any hypothesis are asymptotically equivalent, but that they for any finite sample size obey the inequality LM > LR > Wald. The present paper introduces the Wald test as a device for detecting spatial nonstationarity and derives the finite-sample distribution of this test under the null using Monte Carlo simulation. Though focus is on the application of the test as a device to reveal spatial nonstationarity, the established results can be straightforwadly generalised to obtain a device to test for spurious regression and for spatial cointegration along the lines suggested by Lauridsen and Kosfeld (2004).

2. MODELS WITH SPATIAL DYNAMICS

2.1. The regressive, spatially autoregressive model

The first order spatially autoregressive model (SAR(1) model) was initially studied by Whittle (1954) and has been used extensively in works by Ord (1975); Cliff and Ord (1981); Ripley (1981); Upton and Fingleton (1985); Anselin (1988a); Griffith (1992); Haining (1990); Lauridsen (2004). For applied research the SAR(1) model is extended by explanatory variables (see Upton and Fingleton, 1985; Anselin, 1988a; Haining, 1990; Lauridsen, 2004). The regressive, spatially autoregressive model (SARX(1) model) is established as

$$y = \rho W y + X\beta + \nu , \qquad \textbf{[2.1]}$$

in which **y** is an nx1 vector, **X** an nxK matrix of explanatory variables, ρ the autoregressive parameter, **I** the nxn identity matrix and **v** an nx1 vector of independently normally distributed errors with zero expectation and variances σ^2, i.e. $\boldsymbol{\nu} - N(\mathbf{0},\sigma^2\mathbf{I})$, **W** denotes an nxn spatial weight matrix. It is obtained by row-standardisation of the nxn contiguity matrix **W*** which is defined by $W^*_{ij} = 1$ if the areal units i and j are neighbours, and $W^*_{ij} = 0$ otherwise, i.e. $W_{ij} = W^*_{ij} \,/\, \Gamma_{j=1..n} W^*_{ij}$. For alternative specifications of the spatial weight matrix, see e.g. Cliff and Ord (1981) and Anselin (1988a). **W** may be noncircular, which is the case for the time-series case where $W_{ij} = 1$ if $j = i$-1, for $i = 2,3,..,n$. For the general spatial case, **W** is generally circular. As proved by Anselin (1988a), circularity of **W** renders OLS estimation of the parameters inefficient. Finally, for the general case, ρ is restricted to the interval between -1 and +1 and thus may assume positive as well as negative values. Although meriting interest in itself, the negative case is conceptually different from the usual positive case. We thus narrow our focus in the present investigation to the common case where ρ is positive.

2.2. Spurious regression and nonstationarity

If **y** and one or more of the **x** variables are generated according to SAR scemes with positive autoregressive parameters and **y** is regressed on **X**, i.e.

$$y = X\beta + \varepsilon, \qquad \textbf{[2.2]}$$

with ε**,** as the error term, a risk of spurious regression occurs. Especially, in the case of spatial nonstationarity, where **y** and one or more of the **x** variables have autoregressive parameters close to 1, the risk of spurious regression is alarmingly high. It manifests in the OLS residuals **e** of the regression tending to be highly spatially autocorrelated. This is demonstrated in Fingleton (1999) where extremely high values of the test statistics of the Moran test for spatial autocorrelation (Whittle, 1954; Anselin, 1988a) have been found. In this setting high values of Moran's I can be viewed as the counterpart of low values of the Durbin-Watson statistic having been established in spurious time-series regression. In both cases the behaviour of the test statistics is used as an indication of nonstationarity.

The stochastic process that the OLS residual **e** of the regression (2.1) are generated from usually has to be inferred by inspecting their behaviour. Fingleton (1999) leaves it as an open question how to separate the case of stationary positive autocorrelation ($0<\rho<1$) from the nonstationarity case ($\rho=1$). Moreover, Fingleton (1999) does not address the well-known power of the Moran I test towards misspecifications e.g. in the form of spatial heterogeneity (Anselin, 1988a). Being an advantage in some circumstances, this feature of the Moran I is not necessarily an advantage when investigating specific features of the data generating processes underlying the model in consideration.

In order to account for both shortcomings, Lauridsen and Kosfeld (2004) suggested a two-step Lagrange Multiplier test for spatially autocorrelated errors. The LM error statistic (LME) developed in Anselin (1988a, 1988b),

$$\text{LME} = (\mathbf{e'We} \,/\, \sigma^2)^2 \,/\, \text{tr}(\mathbf{W}^2 + \mathbf{W'W}), \qquad \textbf{[2.3]}$$

is asymptotical χ^2 distributed with 1 degree of freedom under $H_0{:}\rho_\varepsilon = 0$. Therefore, a large LME value indicates either spatial nonstationarity or stationary, spatial error autocorrelation. This result corresponds to the suggestions of Fingleton (1999) with the Moran I test replacing the LM test. Next, under the null of nonstationarity, $H_0{:}\rho_\varepsilon =$ 0, $\rho\varepsilon = \mu \Leftrightarrow \varepsilon = \rho^+\mu$ follows from the spatial error process $\varepsilon = \rho_\varepsilon \mathbf{W}_\varepsilon + \boldsymbol{\mu}$, $\boldsymbol{\mu}\sim N(\mathbf{0},\sigma^2\mathbf{I})$, with $\boldsymbol{\rho} = \mathbf{I} - \mathbf{W}$ as the spatial difference operator. $\boldsymbol{\rho}^+$ denotes the Moore-Penrose generalised inverse which satisfies the conditions $\boldsymbol{\rho}^+\boldsymbol{\rho}\boldsymbol{\rho}^+ = \boldsymbol{\rho}^+$ and $\boldsymbol{\rho}\boldsymbol{\rho}^+\boldsymbol{\rho} = \boldsymbol{\rho}$. By employing the spatial difference operator $\boldsymbol{\rho}$ to (2.2) the transformed regression equation

$$\boldsymbol{\rho}\mathbf{y} = \boldsymbol{\rho}\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\mu} \qquad \textbf{[2.4]}$$

is obtained. Equation (2.4) implies that a regression of $\boldsymbol{\rho}\mathbf{y}$ on $\boldsymbol{\rho}\mathbf{X}$ provides i.i.d. errors, so that the LM error test statistic for this spatially differenced model (DLME) will be close to zero. On the other hand, if the null of nonstationarity, $H_0{:}\ \rho_\varepsilon = 1$, does not hold, then the spatial differencing will bring about an error term of the form $\boldsymbol{\rho}\boldsymbol{\varepsilon} = (\mathbf{I}-\mathbf{W})(\mathbf{I}-\rho_\varepsilon \mathbf{W})^{-1}\boldsymbol{\mu}$, or $\boldsymbol{\mu} = (\mathbf{I}-\rho_\varepsilon \mathbf{W})\ \boldsymbol{\varepsilon}$.

The spatially autocorrelated errors resulting from a spatially “overdifferencing” are expected to go along with a positive DLME value. Concluding, the test strategy consists of calculating and inspecting the LME and the DLME values, leading to one of three conclusions (where the test result is termed to be “positive” if the LM test statistic differs significantly from zero and “zero” otherwise): Nonstationary, spurious regression (LME positive, DLME zero); stationary spatial autocorrelation (LME and DLME positive); or absense of autocorrelation (LME zero, DLME positive).

It is further suggested by Lauridsen and Kosfeld (2004) to investigate whether y or any of the x variables are spatially nonstationary. This may be revealed by using the suggested procedure for a regression of the variable in question (i.e. z being one of y, x_1, x_2, ...) on a constant term. Specifically, the regressions $\mathbf{z} = \boldsymbol{\alpha}\mathbf{i} + \boldsymbol{\varepsilon}$ and $\boldsymbol{\rho}\mathbf{z} = \boldsymbol{\alpha\rho}\mathbf{i} + \boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}$ readily provide the LME and DLME test statistics, which lead to one of three conclusions: **z** is spatially nonstationary (LME positive, DLME zero); **z** represents a stationary SAR scheme (LME positive, DLME positive); or **z** is free of any spatial pattern (LME zero, DLME positive). According to the data generating process $\mathbf{z} = \boldsymbol{\rho}\mathbf{W}\mathbf{z} + \boldsymbol{\nu}$, the **z** variables are spatially integrated of order one, SI(1), in the case of nonstationarity.

An appealing alternative to the LM test procedure suggested is to estimate the SAR model and test the hypothesis $\rho=1$ using a Wald test. This proposal resembles the Dickey-Fuller approach applied to the time series case. However, even for this special case, it is known that $(1-\rho)/\text{s.e.}(\rho)$ does not adhere to a standard normal or *t* distribution under nonstationarity. Thus, it is necessary to know the distribution of the Wald test under spatial nonstationarity for different sample sizes. A further complication is that this distribution may be dependent on the specific contiguity matrix in question. The present study presents benchmark results based on three different tesselations: the bishop, rook and queen tesselations. These three tesselations cover a broad range of empirical contiguity matrices.

2.3. The Wald test

The Wald test is based on maximum likelihood estimation of the model with spatially autocorrelated residuals. Specifically, the log likelihood function for **y** reads

$$L = (2\pi\sigma^2)^{-n/2} \exp(-(\mathbf{y}-\mathbf{X}\boldsymbol{\beta})'\mathbf{A}'\mathbf{A}(\mathbf{y}-\mathbf{X}\boldsymbol{\beta})/(2\sigma^2))\ |\mathbf{A}|$$

with $\mathbf{A} = \mathbf{I} - \rho\mathbf{W}$ (for a detailed derivation, see Anselin, 1988a). Using the first order conditions derived by Anselin (1988a), it is an easy matter to search the interval (-1, 1) for the estimate of ρ that maximises L. Based on the estimate of ρ, estimates for $\boldsymbol{\beta}$ and σ^2 can be calculated analytically. Inserting these estimates in the expected value of the second order conditions, the covariance matrix for the parameters $\boldsymbol{\theta} = (\boldsymbol{\beta}', \rho, \sigma^2)'$ can be calculated (see Anselin, 1988a for details). Formally, the first order conditions read
$dL/d\beta = (\mathbf{y}-\mathbf{X}\boldsymbol{\beta})'\mathbf{A}'\mathbf{A}\mathbf{X}/(\sigma^2) = \mathbf{0}$, or $\boldsymbol{\beta} = (\mathbf{X}'\mathbf{A}'\mathbf{A}\mathbf{X})^{-1}\mathbf{X}'\mathbf{A}'\mathbf{A}\mathbf{y}$;

$dL/d\rho = (\mathbf{y}-\mathbf{X}\boldsymbol{\beta})'\mathbf{A}'\mathbf{W}(\mathbf{y}-\mathbf{X}\boldsymbol{\beta})/(\sigma^2) - tr(\mathbf{A}^{-1}\mathbf{W}) = 0$; and
$dL/d(\sigma^2) = (2B\sigma^2)^{-n/2} \exp(-(\mathbf{y}-\mathbf{X}\boldsymbol{\beta})'\mathbf{A}'\mathbf{A}(\mathbf{y}-\mathbf{X}\boldsymbol{\beta})/(2\sigma^2)) |\mathbf{A}| = 0$, or $\sigma^2 = (\mathbf{y}-\mathbf{X}\boldsymbol{\beta})'\mathbf{A}'\mathbf{A}$ $(\mathbf{y}-\mathbf{X}\boldsymbol{\beta})/n$.

The covariance matrix is calculated by inserting the maximum likelihood estimates in the inverse to the information matrix, i.e. $\mathbf{V} = \mathbf{I}_2^{-1}$, where $\mathbf{I}_2$ is made up of
$\mathbf{I}_{\beta\beta} = \mathbf{X}'\mathbf{A}'\mathbf{A}\mathbf{X}/\sigma^2$,
$\mathbf{I}_{\beta\rho} = \mathbf{0}$,
$\mathbf{I}_{\beta\sigma 2} = \mathbf{0}$,
$\mathbf{I}_{\rho\rho} = tr((\mathbf{W}\mathbf{A}^{-1})^2) + tr((\mathbf{W}\mathbf{A}^{-1})'(\mathbf{W}\mathbf{A}^{-1}))$,
$\mathbf{I}_{\rho\sigma 2} = tr(\mathbf{W}\mathbf{A}^{-1}/\sigma^2)$, and
$\mathbf{I}_{\sigma 2\sigma 2} = n/(2(\sigma^2)^2)$.

Specifying the hypothesis $\rho = 1$ as $\mathbf{R}\boldsymbol{\theta} = \mathbf{q}$, with $\mathbf{R} = (\mathbf{0}'\ 1\ 0)$ and $\mathbf{q} = 1$ gives the Wald test on general form as $W = (\mathbf{R}\boldsymbol{\theta} - \mathbf{q})'(\mathbf{R}\mathbf{V}\mathbf{R}')^{-1} (\mathbf{R}\boldsymbol{\theta} - \mathbf{q})$, which for the specific hypothesis reduces to $Wald = (\rho-1)^2/V_\rho$, with V_ρ being the diagonal element of $\mathbf{V}$ corresponding to ρ.

3. DISTRIBUTION OF THE WALD TEST UNDER SPATIAL NONSTATIONARITY

In this section, the finite-sample distribution of the suggested Wald test will be investigated using Monte Carlo simulation studies. The Monte Carlo design is as follows: For specific sample size n and matrix **W**: Perform 1.000 iterations:

Generate **u** as an N(0,1) series and **x** as U(0,1).
Let $\mathbf{e} = \boldsymbol{\rho}^{+}\mathbf{u}$ and $\mathbf{y} = \mathbf{i} + \mathbf{x} + \mathbf{e}$.
Calculate the Wald test for the hypothesis $\rho = 1$.

Report 1, 5 and 10 percentiles for the Wald test.

To investigate the impact of contiguity matrix type, we make use of the regular bishop, rook and queen type contiguity matrices based on an *r*H*r* board (so that $n = r^2$) with *r* assumed to take the values 5, 10, 15 and 20, and the irregular *n*=275 matrix of the Danish municipalities. The bishop matrix represents a square tesselation with a connectivity of four for the inner fields on the chessboard and one and two for the corner and border fields, respectively. The queen matrix represents an octogonal tesselation with a connectivity of eight for the inner fields and three and five for the corner and border fields. Thus, these tesselations represent extremes for a number of patterns, including the hexagonal tesselation, which is of importance due to its application for empirical maps in vector and raster based GIS (Boots and Tiefelsdorf, 2000; Tiefelsdorf, 2000). Actually, the hexagonal tesselation can be constructed from the queen tesselation by deleting connections from any field to the fields vertically above and below this. Moreover, most empirically observed observed regional structures in spatial econometrics are made up of regions with a connectivity within the range of the rook and queen tesselations.

TABLE 1. EMPIRICAL DISTRIBUTION OF THE WALD TEST FOR SPATIAL NONSTATIONARITY

Matrix:	*Bishop*				*Rook*				*Queen*				*Empirical*	$\chi^2(1)$
n:	25	100	225	400	25	100	225	400	25	100	225	400	275	
Percentiles:														
1 %	11.98	8.74	9.86		7.24	7.17	6.54		7.75	7.43	6.84		5.99	6.63
5 %	7.69	7.35	7.82		4.83	4.91	5.05		5.11	5.30	5.17		4.82	3.84
10 %	6.64	6.68	7.10		4.09	4.27	4.30		4.41	4.43	4.64		4.17	2.71

The results are reported in Table 1. It is seen that the critical limits of the Wald test under spatial nonstationarity are higher than for the $\chi^2(1)$ distribution. Especially, this holds true for the bishop type matrix. For the rook and queen type matrices, the deviations are approximately equal and found to be most pronounced for the 5 and 10 percentile, thus indicating that the Wald test under nonstationarity has a distribution with a thicker right tail than the $\chi^2(1)$ distribution.

4. AN EMPIRICAL ILLUSTRATION

To illustrate the above concepts, we provide an empirical example which were investigated in more details in Lauridsen and Nahrstedt (1999) and Lauridsen (2004). The model is concerned with determination of a regression model for outcommuting ratios as a function of unemployment, participation rate, density of working places and average household size. The data were from a 1994 census for 275 Danish municipalities. The municipality structure is characterised by an average connectivity of 4.59 and a range from 1 to 8, which is within the ranges of the rook and queen matrices used in the Monte Carlo studies above. This example is especially interesting because Lauridsen (2004) estimated a SARX model with a spatial autoregression parameter as high as 0.99 using IV estimation. Other regional studies, e.g. Rey and Montouri (1999) and Kosfeld et al. (2002) report an autoregressive parameter of moderate size. However, the example of a near unit root shows that the case of spatial nonstationarity has to be taken into account in applied econometrics. For a time series model, an autocorrelation parameter of this magnitude would be considered as a safe indication of nonstationarity. It is thus a tempting question whether an alike indication of spatial nonstationarity may be derived for this model. Table 2 presents a brief description of the data.

TABLE 2. VARIABLES USED FOR EMPIRICAL STUDY

Variable	*Definition*	*Mean*	*S.D.*	*Min*	*Max*
OUTCOM	*Number of persons with residence in the municipality and workplace in another municipality in percentage of the number of workplaces in the municipality*[a]	*58.14*	*37.79*	*6.00*	*237.00*
PSH1766	Population share of 17-66 year-olds (%)[a]	65.22	2.85	57.90	74.20
WORKPL	Number of workplaces per 100 inhabitants[a]	43.11	11.63	21.00	100.00
IPHOUS	Number of inhabitants per household[a]	2.39	0.16	1.74	2.77
UNEMP	Number of unemployed per 100 17-66 year-olds[a]	9.37	2.24	5.00	18.70
Proximity matrix:					
W1	Neighbourhood matrix for N=275 Danish municipalities[b]				
	Description of number of links per municipality:	4.59	1.68	1	8
	Density of $\mathbf{W}_1$ = .017				
W	Row standardization of $\mathbf{W}_1$				

Data collected 1994, for N=275 Danish municipalities.
Source: a : Statistics Denmark, Copenhagen.
b : Own construction.

Table 3 presents the ML estimation of the model. In Lauridsen (2004) it was left as an open question whether the unexpected negative sign for the UNEMP coefficient was caused by spuriosity due to spatial nonstationarity, see alsoLauridsen and Nahrstedt (1999). The Wald tests for spatial nonstationarity, provided in Table 3, point to stationarity of the residuals as well as of the single variables. An alike conclusion was derived by Lauridsen and Kosfeld (2004) based on OLS estimation and LM tests. It is thus safely concluded that the single variables as well as the entire regression are stationary. Thus, the negative sign for unemployment is rather due to structural properties than to spatial nonstationarity.

TABLE 3. ESTIMATION OF COMMUTING MODEL

Dependent variable: OUTCOM.

Variable	Parameter	Standard Error	T value	Probability
Intercept	-245.17	35.72	-6.86	<.001
UNEMP	-3.58	0.58	-6.15	<.001
PSH1766	4.72	0.43	10.98	<.001
WORKPL	-2.23	0.09	-25.59	<.001
IPHOUS	52.68	7.79	6.76	<.001
ρ_ε	0.63	0.05	11.51	<.001

Tests for nonstationarity:

Variable	Wald	Prob($\Pi^2(1)$)	Prob(Empirical)
OUTCOM	41.83	<0.01	<0.01
UNEMP	16.17	<0.01	<0.01
PSH1766	10.08	<0.01	<0.01
WORKPL	34.61	<0.01	<0.01
IPHOUS	8.26	<0.01	<0.01
residual	47.26	<0.01	<0.01

5. CONCLUSIONS

Until recently, it has not been well established how to separate the case of spatial nonstationarity from the case of stationary positive autocorrelation. As a consequence, reliable diagnostics for spurious spatial regression and for the existence of spatial cointegrating relations have not been available. The present study contributes to close these gaps by proposing a Wald test for detecting spatial nonstationarity. By means of Monte Carlo simulations the finite sample distribution of the suggested Wald test is provided for a fairly general set of contiguity matrix types under varying finite sample sizes. It is found that the critical values for the Wald test for nonstationarity are generally higher than the χ^2 critical values.

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