A Stylized Applied Energy-Economy Model for France

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ABSTRACT

We build, calibrate and simulate a stylized energy-economy model designed to evaluate the magnitude of carbon tax that would allow the French economy to reduce by a factor of four its CO_2 emissions at a forty-year horizon. We estimate the substitution possibilities between fossil energy and other factors for households and firms. We build two versions of the model, the first with exogenous technical progress, and the second with an endogenization of the direction of technical progress. We show that if the energy-saving technical progress rate remains at its recent historical value, the magnitude of the carbon tax is quite unrealistic. When the direction of technical progress responds endogenously to economic incentives, CO_2 emissions can be reduced by more than that allowed by the substitution possibilities, but not by a factor of four. To achieve this, an additional instrument is needed, namely a subsidy to fossil energy-saving research. The redirection of technical progress, which is a driver of energy transition, comes at a small cost in terms of the overall growth rate of the economy.

Keywords: CGE model, Energy, Environment, Carbon tax

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1. INTRODUCTION

"Factor 4", a term coined in France, corresponds to the commitment undertaken in 2003 to reduce by at least 75% French greenhouse gas (GHG) emissions by 2050, compared to the 1990 level. The European Council and the European Parliament have also endorsed this objective, and asserted on numerous occasions and in various documents the need to develop long-term strategies to encourage the transition to a low carbon economy. These unilateral commitments are not sufficient per se to efficiently tackle climate change, but if they prove to be successful, they could prompt other countries to act in turn and unlock international negotiations. However, major uncertainties exist at this stage about the cost and even the feasibility of this objective. The debate on the adequate mix of instruments (market instruments, standards, public investments, R&D efforts, etc.) that should be implemented in order to attain such reductions at a reasonable cost is still ongoing.

Market instruments, based on the increase in the price of fossil fuels, are often presented as promising tools to achieve ambitious GHG reductions, because they are economically efficient. Whatever mix of instruments is chosen, these market instruments nevertheless appear inevitable: a situation with low fossil fuel prices indeed seems incompatible with a significant decrease in their utilization. It may be posited that the necessary price increase will occur naturally due to supply-

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demand factors and that no additional policy is necessary. Even if this happens to be true, which is very unlikely, the question remains: what should the consumer price of fossil fuels be for their consumption to be reduced by a factor of four in the long run?

This question has of course been already addressed. In France, an official commission chaired by Alain Quinet was set up in 2008, with the aim of determining the social value of carbon that should be used by the French government in the cost-benefit analysis for public investments (see Quinet (2009)). The approach adopted was to determine the carbon value that should be applied to the whole economy, so as to achieve a 75% reduction in emissions, i.e. the question addressed above. The commission used the results of simulations performed by three French integrated assessment models, GEMINI-E3 (Vielle & Bernard (1998)), POLES (Criqui et al. (2006)) and IMACLIM-R (Sassi et al. (2010)), which computed the initial level and the time path of the carbon value that would allow European economies to reduce their carbon emissions by a factor of four at a forty-year horizon. GEMINI-E3 is a sectoral Computable General Equilibrium model, POLES an extremely detailed bottom-up model, and IMACLIM-R, a hybrid model. Their level of disaggregation and detail allows them to provide an accurate description of sectoral and even sometimes microeconomic effects. Nevertheless, due to their complexity, it is difficult to understand the precise origin of their results, which vary greatly across the three models. As regards the questions we wish to address here, the three models include assumptions on the substitution possibilities in the different sectors of the economy and on the magnitude of sectoral energy-saving technical progress, which is either exogenous or driven by learning-by-doing effects. These assumptions have a major influence on the results obtained, but the complexity of the models, their large size and in particular their sectoral disaggregation are such that it is impossible to deduce, from these assumptions, information such as the implicit average rate of energy-saving technical progress. However, for given substitution possibilities, energy-saving technical progress naturally decreases the carbon value necessary to achieve the emission-reduction objective. It is therefore very important to start with an accurate estimate of the substitution possibilities, and then to disentangle clearly the role of the instrument from that of technical progress to achieve this objective.

We build here a stylized macroeconomic model, sufficiently aggregated so as to ensure that assumptions about technical progress are explicit and their influence can be easily analyzed. We model an open economy producing a generic good, which can be consumed or invested, and importing fossil fuel as its sole source of energy.[1] Whereas, usually, energy is only considered to be an input in the production process, we also introduce here households' consumption of fossil fuels, and the fact that fossil fuels are used together with durable goods. This consumption includes residential energy and fuel for transport. Transport and, to a lesser extent, housing sectors are indeed the larger emitters and have been until now unable to reduce their GHG emissions in France (see Table 1). Both rely heavily on fossil fuel and it seems important to take them properly into account.

Final or intermediate fossil energy consumption can be reduced either by substitutions triggered by an increase in the consumer energy price or by technical progress. Substitution possibilities exist between energy, durable goods and non-durable goods on the households' side, and between energy, capital and labor on the production side. However, these substitution possibilities are limited. The other option is to rely on fossil energy-saving technological progress. Therefore, we introduce two forms of technical progress, respectively labor-saving and energy-saving. The energy-saving technical progress we consider consists of both improvements in energy efficiency

1. In 2009, fossil energy represented 67.5% of total final energy consumption in France, electricity 23.7% and renewables 8.8% (SOes, Bilan de l'énergie 2009).

and the replacement of fossil fuels by renewables. Thus, we do not explicitly introduce renewables in the model.

Table 1: CO_2 Emissions due to Energy, France, 2009 (CSV)

Sector	Mt CO_2	%	Evolution between 1990 and 2009, %
Transport	141	40.2	+ 15.2
Housing-tertiary	92	26.2	−3.7
Industry (non-energy)	61	17.4	−28.5
Agriculture	10	2.8	−0.7
Energy	47	13.4	−22.1
	351	100.0	−6.1

Source: SOes, Bilan de l'énergie 2009

We present two versions of the model.

In Section 2, we develop the first version, with exogenous technical progress. The rates of labor-saving and energy-saving technical progress are estimated using French annual historical data. We address the following question: considering that the rates of technical progress remain those observed in the (recent) past, what carbon price path will enable CO_2 emissions to be reduced by a factor of four within 40 years?[2] The implicit assumption is that the policies put in place in our simulations, namely the increase in fossil fuel consumer prices, have no impact on the rate of fossil energy-saving technical progress, and that no specific policy aimed at increasing this rate is implemented.

We perform three simulations. In the first, the rate of energy saving technical progress equals the average historical value obtained in the estimate, and we introduce the carbon tax proposed in the Quinet report. It shows that this tax path is far from sufficient to reduce CO_2 emissions by a factor of four at a forty-year horizon. It only yields a 25% reduction in emissions. Hence, we conclude that in large applied models there are more substitution possibilities and/or more energy-saving technical progress than in our model. In the second simulation, we determine what the magnitude of an oil shock would have to be in order to reach the same level of reductions as with the tax, and compare the consequences of this oil shock to those of the carbon tax. In the last simulation, we increase exogenously the rate of fossil energy-saving technical progress sufficiently to reduce emissions by a factor of four with the carbon tax recommended in the Quinet report. The rate of fossil energy-saving technical progress must be greatly (unreasonably) increased to reach Factor 4. This exercise remains unsatisfactory since this increase in the technical progress rate is costless, and does not occur at the expense of the other rate of technical progress in the model, the labor-saving technical progress.

In Section 3, we incorporate an endogenous mechanism into the model, so that the rate of technical progress on fossil energy can be stimulated by a price effect and by a size effect of the research effort directed at saving fossil energy. The rate of technical progress associated with energy use is indeed likely to be closely correlated with the level of the energy price. Technical progress is not fully endogenized: the total amount of resources devoted to research is exogenous. We analyze the extent to which the endogenization of the direction of technical change affects the results obtained in the first exercise.

2. CO_2 emissions remained stable between 1990 and 2007. The financial crisis which began in 2008 brought a 6 % reduction (in 2009) compared to 1990 levels. As the model developed here is a long term model, we do not account for short term fluctuations and consider that the level of emission in 2010 is the same as in 1990, so that the Factor 4 objective consists in dividing emissions by four from 2010 to 2050.

We perform similar simulations as in the previous section. The results are as follows. When the direction of technical progress is endogenous, the introduction of the carbon tax induces a re-direction of the research effort towards energy-saving technical progress. Its rate immediately increases greatly, and stabilizes in the medium run above its baseline value. It comes at a small cost in terms of overall growth. Nevertheless, the re-direction of technical progress is not sufficient to reach the Factor 4 objective. A supplementary measure is needed, namely a subsidy to fossil energy-saving technical progress.

2. THE MODEL WITH EXOGENOUS TECHNICAL PROGRESS

The first version of the model consists in a standard exogenous growth model integrating fossil fuel use both on the households and firms' side, rigidities in the adjustment of the housing and the productive sectors, and two types of technical progress, respectively labor and energy-saving. We describe successively households' and firms' behaviour and the closure of the model, the calibration method and results, and the simulations performed.

2.1. Households

Several macroeconomists have emphasized that distinguishing non-durable and durable goods is important to obtain an accurate representation, both on a theoretical and an empirical point of view, of households' consumption and savings decisions along their life cycle. Ogaki & Reinhart (1998) for instance show that introducing separately non-durable and durable goods modifies very significantly the estimation of the intertemporal elasticity of substitution of consumption. More recently, Fernandez-Villaverde & Krueger (2011) survey the empirical literature and also conclude that the distinction is meaningful. These papers do not distinguish households' energy consumption from the consumption of other non-durable goods. We think that separating energy consumption considerably reinforces the importance of distinguishing between non-durable and durable goods, because durables almost only need energy to deliver their services whereas non-durables do not. These two types of goods are very different to that respect.

We thus consider that households have access to three types of goods: non-durable goods N, energy (fossil fuels) E and durable goods D. Non-durable goods are consumed during the period, whereas durable goods can be stored or have a long lifespan. Contrary to non-durables and energy, durable goods follow an accumulation process of the standard form:[3]

$$D_t = (1-\delta_d)D_{t-1} + X_t \tag{1}$$

where X_t represents the investment in durable goods at period t and δ_d is the rate of depreciation.

Utility at period t is a function of the consumptions of non-durable goods N_t and energy $E_{h,t}$ in that period, and of the services provided by the stock of durable goods D_{t-1} at the beginning of the period, these services being supposed to be proportional to the stock itself:

$$U(N_t, D_{t-1}, E_{h,t}) = U(C_t)$$

3. The model is simulated with the Dynare software (Adjemian et al. (2011)), which adopts the convention that for stock variables the default is to use a *stock at the end of the period*. Our definition of D_t follows: D_t represents the stock of durable goods at the end of period t, i.e. that will be used by households in the following period $t+1$. The same convention will apply for productive capital and other stock variables.

where C_t is defined by:

$$C_t = \left(\gamma N_t^{\frac{\omega-1}{\omega}} + (1-\gamma) Z_{h,t}^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} \tag{2}$$

$Z_{h,t}$ being a CES aggregate of services provided by durables goods D_{t-1} and efficient energy consumption $A_t^e E_{h,t}$:

$$Z_{h,t} = \left(\nu D_{t-1}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)(A_t^e E_{h,t})^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

A^e represents energy efficiency.

Changing the stock of durable goods induces adjustment costs. We make the assumption that these costs are nil along a balanced growth path, so that households bear them only if they have to deviate from the "normal" trajectory of the economy. These costs are classically specified as:

$$AC_{h,t} = \frac{\kappa_d}{2} \left(\frac{D_t}{(1+g_t^{al})D_{t-1}} - 1 \right)^2 (1+g_t^{al}) D_{t-1} \tag{4}$$

where g^{al} is the growth rate of labor productivity, which will turn out to be the long term growth rate of the economy.[4]

At each period, the representative household can buy or sell bonds which pay or cost a nominal rate r_t. We denote A_{t-1} the nominal value of bonds possessed at the beginning of period t. Households revenues also consist in labor revenues ($P_t^l L_t$) and lump-sum transfers from the government T_t.

The budget constraint at period t reads, with obvious notations for the various prices:

$$(1+\tau_t^c)(P_t^n N^t) + (P_t^e + \tau_{h,t}) E_{h,t} + P_t^x AC_{h,t} + A_t = P_t^l L_t + T_t + (1+r_t) A_{t-1} \tag{5}$$

where τ^c is the tax rate on the consumption of goods and τ_h the additive tax on households' energy consumption.

The representative household seeks to maximize the discounted sum of its utilities under the intertemporal budget constraint:

$$\max \sum_{t=1}^{\infty} \frac{1}{(1+\mu)^t} U(C_t)$$

$$\text{s.c.} \quad \sum_{t=1}^{\infty} \frac{(1+\tau_t^c)(P_t^n N_t + P_t^x X_t) + (P_t^e + \tau_{h,t}) E_{h,t} + P_t^x AC_{h,t}}{\Pi_{s=1}^t (1+r_s)} = A_0 + \sum_{t=1}^{\infty} \frac{P_t^l L_t + T_t}{\Pi_{s=1}^t (1+r_s)}$$

where $\mu > 0$ is the discount rate. The no-Ponzi condition[5] reads:

$$\lim_{t \to \infty} \frac{A_t}{\Pi_{s=1}^t (1+r_s)} = 0$$

4. In this first version of the model, g^{al} is exogenous and constant. We will endogenize it in the second version of the model, where it will not be constant anymore. This is why we keep here the time index.

5. We do not impose explicitly in Dynare the no-Ponzi condition, but simply verify that it is satisfied in the simulations.

We choose a logarithmic utility function:

$$U(C_t) = \ln C_t$$

Let P_t^d be the user cost of the stock of durable goods:

$$P_t^d = P_t^x\left\{(1+\tau^c)\left[\frac{1+r_t}{1+\pi_t^x}-(1-\delta_d)\right]+\kappa_d\left[-\frac{1}{2}(1+g_t^{al})\left(\left(\frac{D_t}{(1+g_t^{al})D_{t-1}}\right)^2-1\right)\right.\right.$$
$$\left.\left.+\frac{1+r_t}{1+\pi_t^x}\left(\frac{D_{t-1}}{(1+g_{t-1}^{al})D_{t-2}}-1\right)\right]\right\} \quad (6)$$

with $\pi_t^x = \dfrac{P_t^x}{P_{t-1}^x}-1$.

The FOC can be written as one forward-looking inter-temporal arbitrage and two static arbitrages between the three consumption goods:

$$\frac{1}{1+\mu}\left(\frac{N_t}{N_{t+1}}\right)^{\frac{1}{\omega}}\left(\frac{C_t}{C_{t+1}}\right)^{\frac{\omega-1}{\omega}} = \frac{1+\pi_{t+1}^n}{1+r_{t+1}} \quad (7)$$

$$\frac{1-\nu}{\nu}\frac{D_t}{E_{h,t+1}}\left(\frac{A_{t+1}^e E_{h,t+1}}{D_t}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \frac{P_{t+1}^e+\tau_{h,t+1}}{P_{t+1}^d} \quad (8)$$

$$\frac{(1-\gamma)(1-\nu)}{\gamma}\frac{N_t}{E_{h,t}}\left(\frac{Z_{h,t}}{N_t}\right)^{\frac{\omega-1}{\omega}}\left(\frac{A_t^e E_{h,t}}{Z_{h,t}}\right)^{\frac{\epsilon-1}{\epsilon}} = \frac{P_t^e+\tau_{h,t}}{(1+\tau_t^c)P_t^n} \quad (9)$$

2.2. Firms

Firms are perfectly competitive. They produce the generic good using capital, labor and fossil fuels, according to the following specification:

$$Y_t = \left[\alpha(A_t^l L_t)^{\frac{\rho-1}{\rho}} + (1-\alpha)Z_{f,t}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \quad (10)$$

$$Z_{f,t} = \left[\beta K_{t-1}^{\frac{\sigma-1}{\sigma}} + (1-\beta)(A_t^e E_{f,t})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \quad (11)$$

$$K_t = (1-\delta_k)K_{t-1} + I_t \quad (12)$$

As for the stock of durable goods, changing the stock of capital leads to adjustment costs, specified as:

$$AC_{f,t} = \frac{\kappa_k}{2}\left(\frac{K_t}{(1+g_t^{al})K_{t-1}} - 1\right)^2 (1+g_t^{al})K_{t-1} \tag{13}$$

The problem of the representative firm reads:

$$\max V_0 = \sum_{t=1}^{\infty} \frac{P_t^y Y_t - P_t^l L_t - P_t^i (I_t + AC_t) - (P_t^e + \tau_{f,t})E_{f,t}}{\Pi_{s=1}^{t}(1+r_s)}$$

s.c. (12), (13)

with τ_f the tax on firms' energy consumption, possibly different from the tax τ_h paid by households.

Let P_t^k be the user cost of capital:

$$P_t^k = P_t^i\left[\frac{1+r_t}{1+\pi_t^i} - (1-\delta_k) + \kappa_k\left[\frac{1+r_t}{1+\pi_t^i}\left(\frac{K_{t-1}}{(1+g_{t-1}^{al})K_{t-2}} - 1\right) - \frac{1}{2}(1+g_t^{al})\left(\left(\frac{K_t}{(1+g_t^{al})K_{t-1}}\right)^2 - 1\right)\right]\right] \tag{14}$$

FOC simply state that the marginal productivity of the inputs is equal to their real cost:

$$\alpha (A_t^l)^{\frac{\rho-1}{1}}\left(\frac{Y_t}{L_t}\right)^{\frac{1}{\rho}} = \frac{P_t^l}{P_t^y} \tag{15}$$

$$(1-\alpha)\left(\frac{Y_t}{Z_{f,t}}\right)^{\frac{1}{\rho}}(1-\beta)(A_t^e)^{\frac{\sigma-1}{\sigma}}\left(\frac{Z_{f,t}}{E_{f,t}}\right)^{\frac{1}{\sigma}} = \frac{P_t^e + \tau_{f,t}}{P_t^y} \tag{16}$$

$$(1-\alpha)\left(\frac{Y_t}{Z_{f,t}}\right)^{\frac{1}{\rho}}\beta\left(\frac{Z_{f,t}}{K_{t-1}}\right)^{\frac{1}{\sigma}} = \frac{P_t^k}{P_t^y} \tag{17}$$

2.3. Government

The government receives tax receipts and reimburses them lump sum to households, so that its budget is balanced at each date:

$$\tau_t^c(P_t^n N_t + P_t^x X_t) + \tau_{h,t}E_{h,t} + \tau_{f,t}E_{f,t} = T_t \tag{18}$$

2.4. Closure

The equilibrium on the generic good market and the labor market respectively read:

$$Y_t = N_t + X_t + I_t + AC_{h,t} + AC_{f,t} + EX_t \tag{19}$$

$$L_t = \overline{L} \tag{20}$$

Note that adjustment costs are costs in terms of the generic good. Exportations of the generic good are denoted EX_t. They are proportional to an exogenous foreign demand for the good $\overline{D_t}$, and respond to a relative price effect:

$$EX_t = \overline{D_t}\left(\frac{er_t\overline{P_t}}{P_t^y}\right)^{-\xi} \tag{21}$$

where $\overline{P_t}$ is the exogenous price of the generic good in the rest of the world in foreign currency, er_t the exchange rate and ξ the price elasticity of exports.

Fossil fuels are totally imported. We do not model the extraction behavior of producers and consider that the producer price in foreign currency $\overline{P}_t^e$ is exogenous and grows at a constant rate, in order to reflect the increasing scarcity of non-renewable resources and to mimic a Hotelling-type behaviour. The price of fossil fuels in domestic currency is $P_t^e = er\overline{P}_t^e$. er_t adjusts at each date as to ensure the equilibrium of the trade balance:

$$P_t^y EX_t = P_t^e(E_{h,t} + E_{f,t}) \tag{22}$$

Starting from the household's budget constraint (5) and using successively the government's budget constraint (18), the zero profit condition of firms, the equilibrium condition on the good market (19), the expressions of I and AC_f in terms of the capital stock K(equations (12) and (13)) and the expression of the user cost of capital (14), we obtain:

$$A_t-(1+r_t)A_{t-1} = P_t^i\left(1+\kappa_k\left(\frac{K_t}{(1+g_t^{al})K_{t-1}}-1\right)\right)K_t - P_t^i\frac{1+r_t}{1+\Pi_t^i}\left(1+\kappa_k\left(\frac{K_{t-1}}{(1+g_{t-1}^{al})K_{t-2}}-1\right)\right)K_{t-1}$$

This equation, together with the initial condition[6] $A_0 = P_0^i K_0$, allows us to obtain the relationship between household's financial wealth and the stock of capital at each date $t>0$:

$$A_t = P_t^i\left(1+\kappa\left(\frac{K_t}{(1+g_t^{al})K_{t-1}}-1\right)\right)K_t \tag{23}$$

The model in standard variables is composed of equations (1) to (22). We choose the production price as numeraire: $P_t^y = 1\ \forall t$. Hence $P_t^n = P_t^i = P_t^x = 1$.

2.5. Long Term

In this version of the model, the growth rates of labor productivity g^{al} and of energy efficiency g^{ae} are exogenous and constant. The price of energy in foreign currency, and hence its growth rate, are exogenous.

We want to describe an economy evolving in the long run along a balanced growth path.[7] The common growth rate of the real economic variables, including efficient energy demands, is

6. This condition is satisfied at the steady state, see below.

7. Indeed, it is possible to perform numerical simulations of the model only if the final state of the economy is a steady state. For that, the model must be written in stationary variables, i.e. in variables deflated by labor in efficiency units $A_t^l L_t$.

necessarily g^{al}, the exogenous rate of labor-augmenting technical progress. Energy efficiency growing at rate g^{ae}, gross energy demands have to grow at rate $(1+g^{al})/(1+g^{ae})-1$. Prices (and the exchange rate) are stationary except for P^l and P^e, which respectively grow at rates g^{al} and g^{ae}. Moreover, the fact that A, T, $\tau_h E_h$ and $\tau_f E_f$ must grow in the long run at rate g^{al} yields that taxes on energy τ_h and τ_f have to grow at rate g^{ae}. Finally, foreign demand $\overline{D}$ must grow at rate g^{al}.

The requirement that the economy evolves along a balanced growth path in the long run is thus very restrictive. The energy price, in foreign currency and in domestic currency, must grow at the same rate than energy efficiency, which suggests introducing an endogenous technical progress induced by the energy price. Taxes on energy must also grow at this same rate.[8] Fossil energy consumption may decrease or increase, depending on the respective magnitudes of g^{al} and g^{ae}.

2.6. Model in Intensive Variables

We note $x_t = X_t/(A_t^l L_t)$ and $p_t^l = P_t^l/A_t^l$. We normalize $\overline{L}=1$. We introduce new variables which are stationary in the long run: $(A^e e_h)_t = A_t^e e_{h,t}$, $(A^e e_f)_t = A_t^e e_{f,t}$, $(P^e/A^e)_t = P_t^e/A_t^e$, $(\tau_h/A^e)_t = \tau_{h,t}/A_t^e$, $(\tau_f/A^e)_t = \tau_{f,t}/A_t^e$. The equations of the model in intensive variables read:

$$c_t = \left(\gamma n_t^{\frac{\omega-1}{\omega}} + (1-\gamma) z_{h,t}^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}} \tag{E1}$$

$$z_{h,t} = \left(\nu\left(\frac{d_{t-1}}{1+g_t^{al}}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)(A^e e_h)_t^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{E2}$$

$$\left(\frac{n_t}{n_{t+1}}\right)^{\frac{1}{\omega}}\left(\frac{c_t}{c_{t+1}}\right)^{\frac{\omega-1}{\omega}} = \frac{(1+g_{t+1}^{al})(1+\mu)}{1+r_{t+1}} \tag{E3}$$

$$P_t^d = (1+\tau^c)(r_t+\delta_d) + \kappa_d\left[(1+r_t)\left(\frac{d_{t-1}}{d_{t-2}}-1\right) - \frac{1}{2}(1+g_t^{al})\left(\left(\frac{d_t}{d_{t-1}}\right)^2-1\right)\right] \tag{E4}$$

$$\frac{d_t}{(1+g_{t+1}^{al})(A^e e_h)_{t+1}} = \left(\frac{\nu}{1-\nu}\frac{(P^e/A^e)_{t+1} + (\tau_h/A^e)_{t+1}}{P_{t+1}^d}\right)^{\varepsilon} \tag{E5}$$

$$\frac{n_t^{\frac{1}{\omega}} z_{h,t}^{\frac{1}{\varepsilon}-\frac{1}{\omega}}}{(A^e e_h)_t^{\frac{1}{\varepsilon}}} = \frac{\gamma}{(1-\gamma)(1-\nu)}\frac{(P^e/A^e)_t + (\tau_h/A^e)_t}{1+\tau_t^c} \tag{E6}$$

$$d_t = \frac{(1-\delta_d)d_{t-1}}{1+g_t^{al}} + x_t \tag{E7}$$

$$k_t = \frac{(1-\delta_k)k_{t-1}}{1+g_t^{al}} + i_t \tag{E8}$$

8. Remember that energy taxes are here unit taxes; ad valorem taxes would have to remain constant.

$$y_t = \left(\alpha + (1-\alpha) z_{f,t}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{E9}$$

$$z_{f,t} = \left(\beta \left(\frac{k_{t-1}}{1+g_t^{al}} \right)^{\frac{\sigma-1}{\sigma}} + (1-\beta)(A^e e_f)_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{E10}$$

$$y_t = \alpha^{-\rho} (p_t^l)^{\rho} \tag{E11}$$

$$\frac{y_t}{z_{f,t}} = (1-\alpha)^{-\rho} (P_t^{zf})^{\rho} \tag{E12}$$

$$\frac{k_{t-1}}{(1+g_t^{al}) z_{f,t}} = \beta^{\sigma} \left(\frac{P_t^{zf}}{P_t^k} \right)^{\sigma} \tag{E13}$$

$$\frac{k_{t-1}}{(1+g^{al})(A^e e_f)_t} = \left(\frac{\beta}{1-\beta} \right)^{\sigma} \left(\frac{(P^e/A^e)_t + (\tau_f/A^e)_t}{P_t^k} \right)^{\sigma} \tag{E14}$$

$$P_t^k = \delta_k + r_t + \kappa_f \left((1+r_t) \left(\frac{k_{t-1}}{k_{t-2}} - 1 \right) - \frac{1}{2} (1+g^{al}) \left(\left(\frac{k_t}{k_{t-1}} \right)^2 - 1 \right) \right) \tag{E15}$$

$$y_t = i_t + n_t + x_t + \frac{\kappa_d}{2} \frac{(d_t - d_{t-1})^2}{d_{t-1}} + \frac{\kappa_f}{2} \frac{(k_t - k_{t-1})^2}{k_{t-1}} + ex_t \tag{E16}$$

$$(1+\tau_t^c)(n_t + x_t) + (A^e e_h)_t((P^e/A^e)_t + (\tau_h/A^e)_t) + \frac{\kappa_d}{2} \frac{(d_t - d_{t-1})^2}{d_{t-1}} + a_t = p_t^l + (1+r_t) \frac{a_{t-1}}{1+g_t^{al}} + t_t \tag{E17}$$

$$t_t = \tau_t^c (n_t + x_t) + (A^e e_h)_t (\tau_h/A^e)_t + (A^e e_f)_t (\tau_f/A^e)_t \tag{E18}$$

$$(A^e e)_t = (A^e e_h)_t + (A^e e_f)_t \tag{E19}$$

$$ex_t = \overline{d}_t (er_t \overline{P}_t)^{-\xi} \tag{E20}$$

$$ex_t = (P^e/A^e)_t (A^e e)_t \tag{E21}$$

$$(P^e/A^e)_t = er_t (\overline{P}^e/A^e)_t \tag{E22}$$

$$wal_t = a_t - \left(1 + \kappa_k \left(\frac{k_t}{k_{t-1}} - 1 \right) \right) k_t \tag{E23}$$

Equation (E23) is here just to ensure that the model is well specified: wal_t must be at each date equal to 0.

2.7. Estimations and Calibration

The elasticities of substitution are central parameters which largely influence the simulation results. In the same way, assumptions on the rates of technical progress are determining. Thus, both for households' utility function and firms' production function, we perform an estimation of these elasticities and rates on French data. On the households' side, we estimate the elasticity of substitution between durable goods and energy, together with the average rate of technical progress related to households' energy use. Concerning the elasticity of substitution between non-durable goods and the aggregation of durable goods and energy, we choose a unitary elasticity. On the firms' side, we estimate the elasticities of substitution in each CES function, together with the rates of technical progress on labor and energy.

2.7.1. Elasticities of substitution of households' utility function

Method and results

We choose a unitary elasticity of substitution (ω in equation (2)) between non-durable goods and the aggregation of durable goods and energy. This choice is motivated by Fernandez-Villaverde & Krueger (2011), who use a Cobb-Douglas aggregation between durable and non-durable goods in the households' utility function. They indeed argue that in most cases, estimated elasticities in the literature are not significantly different from one. For instance, Ogaki & Reinhart (1998) find an elasticity of 1.167, not significantly different from one at the 5% level. Contrary to Fernandez-Villaverde & Krueger (2011), our specification is not a two goods—durables and non-durables—utility function, since we also include energy. Though, as Dhawan & Jeske (2008), we extend the result of Fernandez-Villaverde & Krueger (2011) to a utility function between non-durable goods and an aggregate between the stock of durables and energy and choose $\omega = 1$.

Concerning the elasticity of substitution between durable goods and energy (ε), Dhawan & Jeske (2008) find an elasticity of 0.26 for the United States by matching the theoretical volatility of households' energy use to the one observed in the data. We do not rely on their result because contrary to them, we include building in durable stocks, which is likely to impact the value of the elasticity. We perform an estimation of this elasticity, using a cointegration relation as Ogaki & Reinhart (1998). They indeed stress that the long run information identified by the cointegration relation is appropriate when dealing with durable goods because adjustment costs, although significant, do not affect the long run behavior of consumption of durable goods. They add that this method avoids the computation of the user cost of durable goods P^d on the estimation sample, involving an expectation operator which is difficult to deal with.

For the purpose of this estimation, we show in Appendix A that $\frac{(1+\tau_t^c)P_t^x}{(P_t^e+\tau_{h,t})/A_t^e}\left(\frac{X_t}{A_t^e E_t}\right)^{\frac{1}{\varepsilon}}$ is stationary and that the vector $\left[\ln\left(\frac{(1+\tau_t^c)P_t^x}{(P_t^e+\tau_{h,t})/A_t^e}\right), \ln\left(\frac{X_t}{A_t^e E_t}\right)\right]$ is cointegrated with a cointegrating vector $\left[1, \frac{1}{\varepsilon}\right]$. The estimation of the relation gives the elasticity of substitution ε between durable goods and fossil energy, together with the average rate of technical progress related to households' energy use. We obtain $\varepsilon = 0.50$ and $g^{ae} = 1.6\%$ per year. Thus, we find that fossil energy and the services from durables are poorly substitutable. Notice that with a rate of fossil energy-saving technical progress of 1.6% per year, with no economic growth, fossil fuel consumption would be divided only by 1.9 in 40 years! And 87 years would be necessary to reach the Factor 4 objective. If the economy keeps on growing, the decrease in fossil fuel consumption allowed by fossil energy-saving technical progress, if this rate remains the same as in historical data, will be even further from the 75% reduction in 40 years.

Data

We use data from the French national statistics administration (INSEE) for the period 1959–2010. In national accounts, durable goods are composed of furniture and consumers' equipment such as cars, television sets, refrigerators, etc. To include housing in the stock of durables, we build X_t as the sum of consumption of durable goods and of households' investment, which corresponds to housing investment. The price index P_t^x is built using the chained price methodology with elementary price indexes of housing investment and durable goods consumption. For $E_{h,t}$, we cut off fossil energy consumption from consumption by product provided in national accounts. This cannot be performed exactly with the energy split provided, since electricity is considered as a whole. Taking into account the fact that the share of electricity from fossil origin is very small in France, we totally exclude electricity in our computation. The price is built with the chained price methodology. Note that index prices include taxes on consumption. In particular the price index of fossil energy includes taxes on energy. In the simulation section, we provide details on the decomposition of the price between gross price and taxes.

2.7.2. Elasticities of substitution in the production function

Method and results

We follow van der Werf (2008), see Appendix B.

The rate of labor saving technical progress is found to lie between $g^{al} = 1.5\%$ and $g^{al} = 1.6\%$, depending on the data we take. We estimate g^{ae} between 2.4% and 2.7%. As we assume that the rate of fossil energy-saving technical progress is the same for households' consumption and for production, and given that we found a rate of technical progress of 1.6% for households, we retain a uniform rate of energy-saving technical progress $g^{ae} = 2.0\%$. We find an elasticity of substitution of $\sigma = 0.5$ between capital (K) and fossil fuel use for production (E_f). We find an elasticity of substitution of $\rho = 0.5$ between L and Z_f. In order to reinforce the result, we also follow the methodology of Ogaki & Reinhart (1998) to estimate the long term elasticity between K and E_f and find similar results.

We find that fossil energy, capital and labor are rather complements for production. The only way to reduce fossil energy consumption without decreasing production is to increase energy efficiency, thanks to fossil energy-saving technical progress. Note that we find that the rate of fossil energy-saving technical progress is larger than the rate of labor-saving technical progress. As a result, without any intervention, fossil energy use is progressively reduced, but at a small rate: 0.4% per year. With this rate of decline in fossil fuel use, the 75% reduction target would be reached in 347 years.

Data

We use data on labor, labor cost, value-added and price of value-added from INSEE. We use data on the stock of capital from OECD. The user cost of capital is foregone interest plus depreciation minus capital gain. Here the interest rate is the nominal bond rate (IMF), and capital gain is the growth rate of the price of investment in capital from INSEE. In order to have the total stock of energy from fossil fuels, we use data from INSEE on intermediate good consumption. However, the disaggregation of these tables does not allow us to have the total use (and price) of energy from fossil fuels, because gas is aggregated with water and electricity. We use data on gas consumption from CEREN and gas price from pegase (French ministry of sustainable development) in order to reconstitute total consumption of fossil fuel energy. We run the regression from year 1986 (data on gas are only available from this date) to year 2008.

2.7.3. Calibration of the other parameters

The calibration procedure is standard. We choose a rate of time preference $\mu = 3\%$ which, together with $g^{al} = 1.6\%$, corresponds to a steady state annual interest rate of 4.6%. We follow Fernandez-Villaverde & Krueger (2011) for the annual depreciation rate of durable goods: $\delta_d = 9\%$. We use a standard value for the depreciation of productive capital: $\delta_k = 10\%$. We assume that the price elasticity of exports is equal to 0.6, as in Klein & Simon (2010). The average tax on consumption is $\tau^c = 0.12$. Considering the level of taxes on fossil energy in the economy, we have $\frac{\tau_h}{A^e} = 0.77$ and $\frac{\tau_f}{A^e} = 0.26$ (see Appendix C). We use steady state ratios to set the other parameters. These ratios are computed using annual data from national accounts between 1986 and 2008. We take arbitrarily $A_0^e = 1$ and $P_0^e = 1$ at the initial steady state. With g^{ae} equal to 2.7% and 1.6% respectively for firms and households, we obtain the following average ratios: $\frac{d}{A^e e_m} = 37$, $\frac{n}{A^e e_m} = 23$, $\frac{k}{y} = 2.2$, $\frac{y}{A^e e_f} = 33$.

Table 2 summarizes the value of the main parameters.

Table 2: Value of the Main Parameters

μ	δ_d	δ_k	g^{al}	g^{ae}	ε	ω	σ	ρ	ξ	κ_d	κ_k	τ^c
0.03	0.09	0.10	0.016	0.02	0.5	1	0.5	0.5	0.6	0	0	0.12

The calibrated parameters are: $\nu = 0.9913$, $\gamma = 0.7780$, $\alpha = 0.0012$ and $\beta = 0.6876$.

2.8. Simulations: Carbon Taxes for Factor 4

In France, the international community's objective of keeping the average global temperature increase below 2°C in the long run, has been associated since 2003 to a reduction of GHG emissions by a factor of four at a forty-year horizon: the so-called Factor 4. In 2003, President Chirac and his Prime Minister Raffarin actually committed France to reducing emissions by a factor of four by 2050, from their 1990 level. This commitment has been reassessed many times since then ("Stratégie nationale de développement durable" in June 2003, "Plan climat" in July 2004, "Loi de programme fixant les orientations de la politique énergétique" in July 2005, "Grenelle de l'environnement" in 2007). Several modeling exercises have been performed since 2003 to assess the feasibility of Factor 4, and to compute the carbon tax path that would allow the French economy to meet this objective: the Rapport De Boissieu (2006) "Division par quatre des émissions de gaz à effet de serre de la France à l'horizon 2050" in 2005, the Rapport Quinet in 2009, or the Rapport

De Perthuis (2012), "Trajectoires 2020–2050—Vers une économie décarbonée", released very recently.

In the Quinet Report, the proposed values across time for the carbon tax are the following:

Table 3: Tax Scenario, in €/tCO_2 Proposed in the Quinet Report

	2010	2020	2030	2050
Recommended value	32	56	100	200 (150–350)

Source: Rapport Quinet, 2009

This corresponds roughly to an increase of 3.9% and 6.2% per year to reach respectively 150 € and 350 € within 40 years, starting at 32 €. In order to implement these scenarios, we need to link the price of carbon to taxes expressed as a percentage of the fossil fuel energy price before tax, as defined in the model with τ_h and τ_f. For this purpose, we use the emission factor for each fuel, expressed in kg of CO_2 per hl. We can then infer the price of a carbon tax per hl, which we can compare to the price before tax. Table 9 in Appendix C presents the impact of a tax of 32 € per tonne of CO_2, which is the initial level proposed by Quinet. Weighting each value by the relative consumption, a 32 € tax corresponds to an increase of 15% of the price before tax.

The results are proportional for any given level of the tax. Consequently, while 32 € per tonne corresponds to 15% of the price before tax, 100 € leads to around a 50% increase, and 200 € to a 100% increase. Note that these numbers are the same, whether we consider firms or households, since the price before tax is almost the same.

We study three scenarios: the carbon tax of the Quinet Report, an oil shock of equivalent magnitude, and a combination of the Quinet carbon tax and energy-saving technical progress that allows carbon emissions to be reduced by a factor of four within forty years (Factor 4).

The simulations are performed without adjustment costs,[9] on the durable side as well as on the capital side. We expect that this makes it easier to achieve the desired emission reduction objective, since the economy is flexible and can adapt its durable and capital stocks readily to the new energy price.

2.8.1. Method

Long term limitations, which are inherent to this type of models, have important implications for the simulations. On a balanced growth path, all real variables necessarily grow at the same pace. In particular, $A^e E_h$ and $A^e E_f$ grow at the same rate as other real variables, i.e g^{al}. The difference between g^{al} and g^{ae} has significant implications on long term energy consumption. If energy efficiency grows faster than the economy, then in the long run energy use will tend towards zero. On the contrary, if it grows more slowly, then energy use will tend towards infinity. If we refer to our estimation, $g^{ae} > g^{al}$ in the initial steady state. It means that without any intervention, energy use will be gradually and regularly reduced at a rate $(1 + g^{ae})/(1 + g^{al})$, and in the long run, we would have $E_h = E_f = 0$. Thus, we do not analyze the transition between the initial and the final

9. We then increased adjustment costs, up to a speed of convergence of the stock of capital of 2% per year (see Fève et al (2009)), to evaluate how rigidities impact the results. We found that adjustment costs have mainly effects on the dynamics of the stock of durables and on the stock of capital, but adding these costs does not change a lot the results on other variables so that, for the sake of brevity, we do not present the results with adjustment costs here.

steady states, but rather between the initial steady state and the economy after 40 years, i.e., the horizon at which we want to reduce emissions by 75%.

At $t = 0$, the economy is on a balanced growth path. We note $(A^e e_h)_0$ and $(A^e e_f)_0$ the variables corresponding to initial energy consumption by households and firms. On the households' side, we recall that

$$(A^e e_h)_t = \frac{A^e_t E_{h,t}}{A^l_t}$$

From $t = 1$ on, taxes on energy consumption are implemented and the economy deviates from the initial steady state. We want to have $E_{h,40} = E_{h,0}/4$, knowing that A^l and A^e are exogenous and grow at rates g^{al} and g^{ae} respectively. Thus a 75% reduction in E_h implies that

$$(A^e e_h)_{40} = \left(\frac{1+g^{ae}}{1+g^{al}}\right)^{40} \frac{A^e_0 E_{h,0}/4}{A^l_0} = \frac{1}{4}\left(\frac{1+g^{ae}}{1+g^{al}}\right)^{40} (A^e e_h)_0$$

We have the same expression between $(A^e e_f)_{40}$ and $(A^e e_f)_0$. In order to reach these reductions, we simulate the effect of a permanent tax, starting today and proportional to the oil price (so that the tax grows at the same rate as P^e). Moreover, we add, in some simulations, an increase of the rate of technical progress directed toward energy A^e during 40 years, from 2010 until 2050.

2.8.2. Simulation 1: Carbon tax of the Quinet Report

In this first simulation, we simulate the impact of the carbon tax proposed in the Quinet Report. The initial level of this tax is 32 €/tCO_2 in 2010, growing then at a rate of 4% per year. It adds 0.15 to the initial tax (representing, for a price of 1, 0.77 for households and 0.26 for firms) in the model, as explained in Appendix C. The results are presented in Tables 4 and 5. In Table 4, we see that this tax alone is not sufficient to generate a 75% reduction in oil consumption and would only result in a 25% reduction by 2050. This can be attributed to the restricted substitution possibilities and the low rate of fossil energy saving technical progress. Table 4 also presents a measure φ of the welfare gains/losses associated with the policy shock. φ is calculated as the equivalent percentage gain/loss of consumption over forty years. The welfare loss associated with the first simulation is equivalent to a forty-year consumption loss of 0.73 %. The welfare loss is thus relatively small, as is the decrease in fossil fuel consumption.

Details on the impact of this Quinet tax on economic variables over time are presented in the two first columns of Table 5. Initially, the increase in energy prices leads to a decrease in energy consumption by households (E_h) and energy use by firms (E_f). This price shock also results in a decrease in durables (D), as D and E_h are complements. However, as non-durables N and the aggregate composed of durables and energy are more substitutable (the elasticity of substitution equals 1), the initial increase in energy prices leads to an increase in consumption of non-durables via a substitution effect. On the production side, production factors are not very substitutable, so that production falls rapidly. At the end of the shock, after 40 years, all variables are below their baseline value, except for the exchange rate. The fall in energy consumption leads to a decrease in the value of imports. The relative price of domestic goods, compared to foreign goods, increases so that the decrease in exports offsets the decrease in imports.

Table 4: Simulations Results (1), Exogenous Technical Progress

	Simulation 1	Simulation 2	Simulation 3
E_{2050}/E_{2010}	0.74	0.74	0.25
φ (%)	−0.73	−2.03	9.88

Table 5: Simulations Results (2), Exogenous Technical Progress

	Simulation 1		Simulation 2		Simulation 3	
% diff. with baseline	1 year	40 years	1 year	40 years	1 year	40 years
C	0.04	−1.28	−0.70	−3.05	1.80	9.93
N	0.46	−0.13	−0.49	−2.04	1.40	3.84
D	−0.39	−2.66	−1.06	−4.23	3.00	17.86
E_h	−5.55	−12.52	−5.76	−13.16	14.84	−67.86
Y	−0.30	−0.90	−0.28	−0.79	0.58	2.68
E_f	−7.24	−14.18	−6.87	−13.05	17.70	−74.36
er	4.26	9.42	−15.86	−26.17	−44.42	125.96

2.8.3. Simulation 2: Oil shock

We want to know whether an exogenous increase in the producer price of oil would have the same effects as a carbon tax. We simulate the consequences of such a shock, calibrated so as to ensure a 26% reduction in emissions by 2050, for comparability with Simulation 1. Remember that, in the baseline, the exogenous foreign oil price increases at a rate of 2%. We assume that the shock on the oil price is as follows: from date 1 to date 40, the foreign oil price is the sum of the baseline price (increasing at a rate of 2%) and an additional price component increasing at a rate of 4% (as the Quinet tax in Simulation 1). To ensure a 26% reduction in emissions by 2050, the initial value of this additional price component must equal 40% of the baseline price, to be compared to 15% added to the baseline price in Simulation 1.

There are two main differences with Simulation 1: (1) the carbon tax in Simulation 1 provides the government with tax receipts, whereas the oil shock benefits the foreign economy, (2) the exchange rate increases in the case of the carbon tax, whereas it decreases in the case of the oil shock. Because of the exchange rate adjustment, the additional price component needed to achieve a 26% reduction in emissions is larger than with the Quinet tax. Because of the possibility of recycling the tax proceeds in Simulation 1, the welfare loss is larger in Simulation 2, for the same emission reduction. This loss is equivalent to a forty-year decrease of 2.03% in consumption (see column 2 of Table 4).

Details on the impact of this oil shock on economic variables over time are presented in columns 3 and 4 of Table 5. As in the former simulation, the increase in the energy price leads to a decrease in energy consumption by households (E_h) and energy use by firms (E_f). This price shock also entails a decrease in durables (D), as D and E_h are complements. Contrary to Simulation 1, even if non-durables N and the aggregate composed of durables and energy are more substitutable, the initial increase in the energy price leads to a decrease in the consumption of non-durables. This is the case because the revenue effect is larger than the substitution effect. On the production side, production falls but less than in Simulation 1. This can be attributed to the increase in exports (in Simulation 1, there was a decrease in exports). Indeed, the exchange rate decreases in this simulation, contrary to Simulation 1. The increase in the foreign energy price leads to an increase in the value of imports. The relative price of domestic goods, compared to foreign goods, decreases so that the increase in exports offsets the increase in imports.

2.8.4 Simulation 3: Factor 4

Given the results of the first two simulations, the question arises as to how to ensure emissions can be reduced by a factor of four. There are two ways in the model to achieve greater reductions: increase taxes or increase the rate of energy-saving technical progress. We find that the initial level of the carbon tax that would allow the economy to achieve a 75% reduction in emissions would be obviously too high to be acceptable (i.e. +3.9 instead of +0.15, or 832 € per tonne of

CO_2 instead of 32 €). We then run a third simulation, aimed at determining the rate of technical progress necessary to achieve Factor 4: we simulate the impact of the carbon tax in the Quinet Report, as in Simulation 1, associated with an increase in g^{ae}, with this increase being calibrated to ensure that emissions are reduced by a factor of 4 by 2050. This simulation is interesting because assumptions about the energy-saving technical progress introduced in applied models used to assess the effects of climate policy are often specific to sectors of the economy and can vary across time. It is therefore very difficult to sum up these assumptions to obtain the implicit average growth rate of the energy-saving technical progress in these simulations. We find that this new g^{ae} is equal to 7.4% per year, instead of 2% in the baseline. The results of this simulation (Simulation 3) are presented in the third column of Table 4 and in columns 5 and 6 of Table 5. It is interesting to note that the initial increase of g^{ae} leads to an initial increase in (E_{fh}) and (E_h), due to a rebound effect.

The main conclusion from these simulations is that the target of 75% cannot be reached without additional energy-saving technical progress. In Simulation 3, the energy directed-technical progress is exogenous and free, so that increasing g^{ae} only yields positive benefits. The welfare gain associated with this simulation is equivalent to a 9.88% consumption gain for forty years. We think that this is misleading. That is why we constructed a second version of the model, in which the direction of technical progress is endogenized, so that increasing fossil energy-saving technical progress is costly.

3. THE MODEL WITH DIRECTED TECHNICAL CHANGE

We now model directed technical change, in the sense that an increase of the energy price (due to an exogenous supply shock or an increase in environmental taxation) induces R&D aimed at saving energy, at the expense of R&D aimed at increasing labor productivity. Popp (2004), for instance, provides empirical evidence of this partial crowding-out effect.

We make the assumption that the research effort of the economy is a given proportion of output: we do not endogenize the intensity of this effort.[10] Nevertheless, given the total amount of resources devoted at each date to R&D, we endogenize the *direction* of technical progress, that is the allocation of this amount between an energy research sector enhancing the efficiency of energy and a labor research sector enhancing the efficiency of labor. This direction responds endogenously to economic incentives. It shapes, to a very great extent, the future characteristics of the economy, as there is now a trade-off between economic growth and energy transition. Indeed, a high labor-saving technical progress ensures a high growth rate of the economy but may result in high CO_2 emissions, whereas a high energy-saving technical progress enables the economy to reduce CO_2 emissions by more than substitution possibilities would allow, but would possibly not allow a high overall growth rate to be achieved.

3.1. The Direction of Technical Progress

The intensity of the research effort of the economy (in terms of the final good) is constant, exogenous and denoted ϕ, with $0 < \phi < 1$. $S_t = \phi Y_t$ is the level of the research effort. By construction, it grows at the same rate than Y_t, i.e. at rate g_t^{al}.

We endogenize the allocation of this amount to an "energy research" sector (S_t^e) and to a "labor research" sector (S_t^l). To this purpose, we introduce the share $sh_t = S_t^l / S_t$.

We build on Smulders & de Nooij (2003), and Acemoglu et al (2012).

10. See Hassler et al. (2011) for a similar simplifying assumption

The productive sector is composed of three types of firms: final goods producers, intermediate goods producers and firms doing research. We present successively the three optimization programs.

At any date, final goods producers use an homogeneous stock of capital (K), labour services (Y_L) and energy services (Y_R) to produce final goods (Y). The three inputs are imperfect substitutes, and we adopt the same two-level CES disaggregation than in the first version of the model, with $\sigma = \rho$:

$$Y = \left[\alpha Y_L^{\frac{\rho-1}{\rho}} + (1-\alpha)\left(\beta K_{-1}^{\frac{\rho-1}{\rho}} + (1-\beta) Y_E^{\frac{\rho-1}{\rho}} \right) \right]^{\frac{\rho}{\rho-1}} \tag{24}$$

The price of the final good is normalized to one and the (accounting) prices of labour and energy services are denoted P^{y_L} and P^{y_E}, respectively. Final goods producers maximize profits, taking prices as given. They demand labour services, energy services and all-purpose capital up to the point where the marginal productivity of these inputs equals their cost. This yields:

$$\frac{Y_E}{Y_L} = \left(\frac{(1-\alpha)(1-\beta)}{\alpha} \frac{P^{y_L}}{P^{y_E}} \right)^{\rho} \tag{25}$$

$$K_{-1} = Y_E \left(\frac{\beta}{1-\beta} \frac{P^{y_E}}{P^k} \right)^{\rho} \tag{26}$$

The demand for energy services relative to labor services is a function of their relative prices.

In a second stage, the services of labor (energy) are obtained by combining[11] raw labor L (raw fossil energy E) and a continuum of sector-specific intermediates x_j^l (x_j^e), of quality A_j^l (A_j^e):

$$Y_L = L^{1-\lambda} \int_0^1 (A_j^l)^{1-\lambda} (x_j^l)^{\lambda} dj \tag{27}$$

$$Y_E = E_f^{1-\lambda} \int_0^1 (A_j^e)^{1-\lambda} (x_j^e)^{\lambda} dj \tag{28}$$

Thus we suppose that there exist three types of machines: all-purpose homogeneous machines, which total stock is K, and specialized sector-specific machines x_k^i.[12] All those machines are produced using the final good only. The interpretation of these sector-specific intermediates is the following. For the production of labour services, these specialized machines are mainly machines embodying ITC. For the production of energy services, they can be either devices aimed at

11. We suppose that the parameter characterizing this combination, λ, is the same for labor and energy services. This assumptions does not have any theoretical or empirical basis and is only made for simplicity, as it is the case in the rest of the literature.

12. As it is the case in the rest of the literature, again, we treat these sector-specific machines as a flow and not a stock, which would be obviously a better assumption.

improving the energy efficiency of existing capital, or specialized capital allowing the production of renewable energy, like solar cells or wind mills.

Final goods producers choose how to produce labor and energy services. For labor services for instance, they solve the following problem:

$$\max P^{y_L} L^{1-\lambda} \int_0^1 (A_j^l)^{1-\lambda} (x_j^l)^{\lambda} dj - P^l L - \int_0^1 P_j^l x_j^l dj$$

The FOC read:

$$x_j^l = \left(\frac{\lambda P^{y_L}}{P_j^l} \right)^{\frac{1}{1-\lambda}} A_j^l L \qquad (29)$$

$$P^{y_L} Y_L = \frac{1}{1-\lambda} P^l L \qquad (30)$$

and for energy services we obtain equivalently:

$$x_j^e = \left(\frac{\lambda P^{y_E}}{P_j^e} \right)^{\frac{1}{1-\lambda}} A_j^e E_f \qquad (31)$$

$$P^{y_E} Y_E = \frac{1}{1-\lambda} P^e E_f \qquad (32)$$

In a third stage, specialized intermediates are supplied by firms in monopolistic competition—the research sector. Producing one unit of these machines costs c units of the final good. Firms producing intermediates aimed at enhancing labor productivity maximize their profit, taking into account the inverse demand function (30):

$$\max \Pi_j^l = (P_j^l - c) x_j^l$$

s.c. (29)

FOC yield:

$$P_j^l = \frac{c}{\lambda}$$

and profit writes

$$\Pi_j^l = \frac{1-\lambda}{\lambda} c \left(\frac{\lambda^2 P^{y_L}}{c} \right)^{\frac{1}{1-\lambda}} A_j^l L$$

Hence

$$Y_L = \left(\frac{\lambda^2 P^{y_L}}{c}\right)^{\frac{\lambda}{1-\lambda}} A^l L \tag{33}$$

with

$$A^l = \int_0^1 A_j^l dj$$

the average productivity of specialized inputs, and, using the FOC on raw labour (30):

$$P^{y_L} = \frac{1}{(1-\lambda)^{1-\lambda}\lambda^{2\lambda}}\left(\frac{P^l}{A^l}\right)^{1-\lambda} c^{\lambda} \tag{34}$$

The accounting price of labour services is a Cobb-Douglas combination of the effective price of labour in efficiency units and of the unit production cost of intermediates.

We obtain equivalent equations for the other research sector:

$$P_j^e = \frac{c}{\lambda},\ \pi_j^e = (1+\tau^r)\frac{1-\lambda}{\lambda} c\left(\frac{\lambda^2 P^{y_E}}{c}\right)^{\frac{1}{1-\lambda}} A_j^e E_f$$

with τ^r the subsidy to the energy research sector, which we introduce as a new potential economic policy instrument, and

$$Y_E = \left(\frac{\lambda^2 P^{y_E}}{c}\right)^{\frac{\lambda}{1-\lambda}} A^e E_f \tag{35}$$

$$P^{y_E} = \frac{1}{(1-\lambda)^{1-\lambda}\lambda^{2\lambda}}\left(\frac{P^e + \tau_f}{A^e}\right)^{1-\lambda} c^{\lambda} \tag{36}$$

Dividing (36) by (34) yields:

$$\frac{P^{y_E}}{P^{y_L}} = \left(\frac{P^e + \tau_f}{P^l}\frac{A^l}{A^e}\right)^{1-\lambda} \tag{37}$$

The relative price of energy and labor services depends positively on the relative price of raw inputs and also on the relative productivities of the two types of specialized machines. The higher the relative productivity of fossil energy-saving specialized inputs, the lower the relative price of energy services.

Dividing (35) by (33) yields:

$$\frac{Y_E}{Y_L} = \left(\frac{P^{y_E}}{P^{y_L}}\right)^{\frac{\lambda}{1-\lambda}} \frac{A^e E_f}{A^l L} \tag{38}$$

Eliminating Y_E/Y_L between (25) and (38) yields:

$$\frac{A^e E_f}{A^l L} = \left(\frac{(1-\alpha)(1-\beta)}{\alpha}\right)^{\rho}\left(\frac{P^{y_L}}{P^{y_E}}\right)^{\rho + \frac{\lambda}{1-\lambda}} \tag{39}$$

Replacing in this equation P^{y_L} and P^{y_E} by their expressions in (34) and (36) yields:

$$\frac{A^e E_f}{A^l L} = \left(\frac{(1-\alpha)(1-\beta)}{\alpha}\right)^{\rho}\left(\frac{P^l/A^l}{P^e/A^e}\right)^{(1-\lambda)\rho + \lambda} \tag{40}$$

As in Acemoglu et al. (2012), productivities evolve according to:[13]

$$A_t^l = (1 + \gamma_L \eta_L sh_{t-1})A_{t-1}^l \tag{41}$$

$$A_t^e = (1 + \gamma_E \eta_E (1 - sh_{t-1}))A_{t-1}^e \tag{42}$$

where γ_L and γ_E are the sizes of an innovation in each research sector, sh the normalized research effort in the research sector aimed at enhancing labor productivity, η_L the probability of success in this research sector, and η_E the probability of success in the other research sector.[14] These equations read equivalently:

$$g_t^{al} = \gamma_L \eta_L sh_{t-1} \tag{43}$$

$$g_t^{ae} = \gamma_E \eta_E (1 - sh_{t-1}) \tag{44}$$

The expected profit of research aimed at enhancing labor productivity is:

$$\Pi_t^l = \eta_L (1 + \gamma_L)\frac{1-\lambda}{\lambda} c \left(\frac{\lambda^2 P_t^{y_L}}{c}\right)^{\frac{1}{1-\lambda}} L_t A_{t-1}^l$$

and we have an equivalent expression for Π_t^e. Dividing both equations yields:

$$\frac{\Pi_t^l}{\Pi_t^e} = \frac{1}{1+\tau_t^r}\frac{(1+\gamma_L)\eta_L}{(1+\gamma_E)\eta_E}\left(\frac{P_t^{y_L}}{P_t^{y_E}}\right)^{\frac{1}{1-\lambda}}\frac{L_t}{E_{f,t}}\frac{A_{t-1}^l}{A_{t-1}^e} \tag{45}$$

13. We do not adopt exactly the same timing as in Acemoglu et al. (2012). We make the assumption, which we find plausible, that the research effort of period $t-1$, and not of period t, determines the productivity level of period t.

14. Note that the model can be also seen as an adoption model instead of a research model. In this case, France does not develop new technologies but adopts existing technologies from other countries. Adoption is costly. In order to incorporate a new intermediate good into the production process, it is necessary to invest resources: η is the probability of succeeding in adapting existing technologies to French production process and sh the relative spending to buy new patents from foreign countries. See Grossman & Helpman (1991).

Acemoglu et al. (2012) identify 3 effects in this relationship, shaping the incentive to innovate in labor-saving technologies versus fossil energy-saving technologies: the direct productivity effect (captured by the term A^l_{t-1}/A^e_{t-1}), which pushes towards innovating in the sector with higher productivity; the price effect (captured by the term $(P^{y_L}/P^{y_E})^{1/(1-\lambda)}$), encouraging innovation toward the sector with higher prices; the market size effect (captured by the term L/E_f), encouraging innovation in the sector with the larger market for specialized inputs.

Replacing in (45) P^{y_L}/P^{y_E} obtained in (37) and using (40) to eliminate E_f/L we get:

$$\frac{\Pi^e_t}{\Pi^l_t} = (1+\tau^r_t)\frac{(1+\gamma_E)\eta_E}{(1+\gamma_L)\eta_L}\left(\frac{(1-\alpha)(1-\beta)}{\alpha}\right)^{\rho}\left(\frac{(P^e_t+\tau_{f,t})/A^e_t}{P^l_t/A^l_t}\right)^{(1-\lambda)(1-\rho)}\frac{1+g^{al}_t}{1+g^{ae}_t} \tag{46}$$

An interior solution is characterized by the same opportunities of profit in the two research sectors, i.e., using also (43) and (44):

$$(1+\tau^r_t)\frac{(1+\gamma_E)\eta_E}{(1+\gamma_L)\eta_L}\left(\frac{(1-\alpha)(1-\beta)}{\alpha}\right)^{\rho}\left(\frac{(P^e_t+\tau_{f,t})/A^e_t}{P^l_t/A^l_t}\right)^{(1-\lambda)(1-\rho)}\frac{1+\gamma_L\eta_L sh_{t-1}}{\gamma_E\eta_E(1-sh_{t-1})} = 1$$

The existence of an interior solution requires $0 < sh < 1$, which we will check ex post.

The two research sectors are owned by the representative consumer so that the profits earned by these research sectors are redistributed lump sum to her. Total transfers are now:

$$\begin{aligned} T_t &= \tau^c_t(N_t+X_t)+\tau_{h,t}E_{h,t}+\tau_{f,t}E_{f,t}-\tau^r_t\lambda(P^e_t+\tau_{f,t})E_{f,t}+\lambda P^l_tL_t+(1+\tau^r)\lambda(P^e_t+\tau_{f,t})E_{f,t}-S_t \\ &= \tau^c_t(N_t+X_t)+\tau_{h,t}E_{h,t}+\tau_{f,t}E_{f,t}+\lambda[P^l_tL_t+(P^e_t+\tau_{f,t})E_{f,t}]-S_t \end{aligned} \tag{47}$$

Finally, we make the assumption that there exist perfect knowledge spillovers such that the fossil energy-saving innovations made in the industrial sector perfectly diffuse to the durable goods sector.

3.2. Model in Intensive Variables

Equations (E1) to (E8), (E15), (E19) to (E23) are unchanged. The new equations are:

$$y_t = \left[\alpha y_{L,t}^{\frac{\rho-1}{\rho}} + (1-\alpha)\left[\beta\left(\frac{k_{t-1}}{1+g^{al}_t}\right)^{\frac{\rho-1}{\rho}} + (1-\beta)y_{E,t}^{\frac{\rho-1}{\rho}}\right]\right]^{\frac{\rho}{\rho-1}} \tag{EE1}$$

$$y_t = \alpha^{-\rho}(P^{y_L}_t)^{\rho}y_{L,t} \tag{EE2}$$

$$\frac{\alpha}{(1-\alpha)(1-\beta)}\left(\frac{y_{E,t}}{y_{L,t}}\right)^{\frac{1}{\rho}} = \frac{P^{y_L}_t}{P^{y_E}_t} \tag{EE3}$$

$$\frac{k_{t-1}}{(1+g^{al}_t)y_{E,t}} = \left(\frac{\beta}{1-\beta}\frac{P^{y_E}_t}{P^k_t}\right)^{\rho} \tag{EE4}$$

$$y_{L,t} = \left(\lambda^2 \frac{P_t^{y_L}}{c}\right)^{\frac{\lambda}{1-\lambda}} \tag{EE5}$$

$$P_t^{y_L} = \frac{1}{(1-\lambda)^{1-\lambda}\lambda^{2\lambda}}(p_t^l)^{1-\lambda}c^{\lambda} \tag{EE6}$$

$$y_{E,t} = \left(\lambda^2 \frac{P_t^{y_E}}{c}\right)^{\frac{\lambda}{1-\lambda}}(A^e e_f)_t \tag{EE7}$$

$$P_t^{y_E} = \frac{1}{(1-\lambda)^{1-\lambda}\lambda^{2\lambda}}((P^e/A^e)_t + (\tau_f/A^e)_t)^{1-\lambda}c^{\lambda} \tag{EE8}$$

$$sh_t^* = \frac{1 + \gamma_E\eta_E - (1 + \tau_{t+1}^r)\frac{(1+\gamma_E)\eta_E}{(1+\gamma_L)\eta_L}\left(\frac{(1-\alpha)(1-\beta)}{\alpha}\right)^{\rho}\left(\frac{(P^e/A^e)_{t+1} + (\tau_f/A^e)_{t+1}}{p_{t+1}^l}\right)^{(1-\lambda)(1-\rho)}}{\gamma_E\eta_E + \gamma_L\eta_L(1 + \tau_{t+1}^r)\frac{(1+\gamma_E)\eta_E}{(1+\gamma_L)\eta_L}\left(\frac{(1-\alpha)(1-\beta)}{\alpha}\right)^{\rho}\left(\frac{(P^e/A^e)_{t+1} + (\tau_f/A^e)_{t+1}}{p_{t+1}^l}\right)^{(1-\lambda)(1-\rho)}} \tag{EE9}$$

$$g_t^{al} = \gamma\eta_L sh_{t-1} \tag{EE10}$$

$$g_t^{ae} = \gamma\eta_E(1 - sh_{t-1}) \tag{EE11}$$

$$t_t = \tau^c(n_t + x_t) + (\tau_h/A^e)_t(A^e e_h)_t + (\tau_f/A^e)_t(A^e e_f)_t + \lambda[p_t^l + (P^e/A^e)_t(A^e e_f)_t + (\tau_f/A^e)_t(A^e e_f)_t] - s_t \tag{EE12}$$

$$s_t = \phi y_t \tag{EE13}$$

$$y_t = i_t + n_t + x_t + \frac{\kappa_d}{2}\frac{(d_t - d_{t-1})^2}{d_{t-1}} + \frac{\kappa_f}{2}\frac{(k_t - k_{t-1})^2}{k_{t-1}} + ex_t + \frac{\lambda^2}{1-\lambda}(p_t^l + (P^e/A^e)_t(A^e e_f)_t + (\tau_f/A^e)_t(A^e e_f)_t) + s_t \tag{EE14}$$

$$(P^e/A^e)_t = \frac{1 + \pi_t^e}{1 + g_t^{ae}}(P^e/A^e)_{t-1} \tag{EE15}$$

3.3. Calibration

The elasticities and the parameters that are in common with the first version of the model have the same value in this second version. As for the new parameters, we retain $\lambda = 0.3$, $c = 0.1$, and we assume that the probability of success is the same in both research sectors, i.e. $\eta_L = \eta_E$. The results of the simulations are quite robust with respect to these assumptions.

The truly important assumption is the value given to *sh*. It is crucial because according to the value of *sh*, the split of research between the two sectors will be on one side or the other of the optimal split, which will have major consequences on the welfare effects of the simulations. Dechezleprêtre et al. (2011) suggests $sh = 0.99$ by counting the energy-saving related patents. We perform two sets of simulation. In the first one, we assume that $sh = 0.99$. We find that the reforms we simulate induce a welfare gain, absent any external effect! This means that, given the calibration, the research effort toward fossil energy-saving technologies is too low so that an increase of oil taxes increases welfare, even without any climate change consideration. We believe that this is a little too optimistic. This result is very sensitive to the baseline value of sh. As we have no clue (except for Dechezleprêtre et al. (2011)) on the true value of this parameter and we believe that other patents may have positive effects on energy-saving technologies, we prefer assuming that, in the baseline situation, one cannot increase welfare by adding a uniform tax (or subsidy) on energy. This is an agnostic point of view: we do not know whether, absent any externality, one should increase or decrease current taxation in order to stimulate or deter research toward energy saving technology. This leads us to take $sh = 0.90$ in a second set of simulation. Note that the choice of the initial sh has a large effect on welfare gains associated with the simulations, but other economic variables, such as production, investment, consumption of durables and energy do not vary a lot when changing the calibration of sh. In particular, the reform always goes with a decrease in GDP.

3.4. Simulations

The growth rate of energy efficiency g^{ae} and the deflator of intensive variables g^{al} are now endogenous (and non-constant). The exogenous variables are intensive variables, $(\tau_f/A^e)_t$. We simulate a shock on $(\tau_f/A^e)_t$. It gives a path for g_t^{ae}, from which we deduce the path $\tau_{f,t}$. We iterate until we obtain the initial value and the time profile we want for $\tau_{f,t}$. We perform four simulations: the first two are identical to the ones in the previous section (carbon tax in the Quinet report and an oil shock), the third one incorporates an increase in the R&D subsidy, and the last one consists of a carbon tax enabling Factor 4 to be reached.

3.4.1. Simulation 1: Carbon tax of the Quinet Report

Table 6: Simulations Results, Endogenous Technical Progress

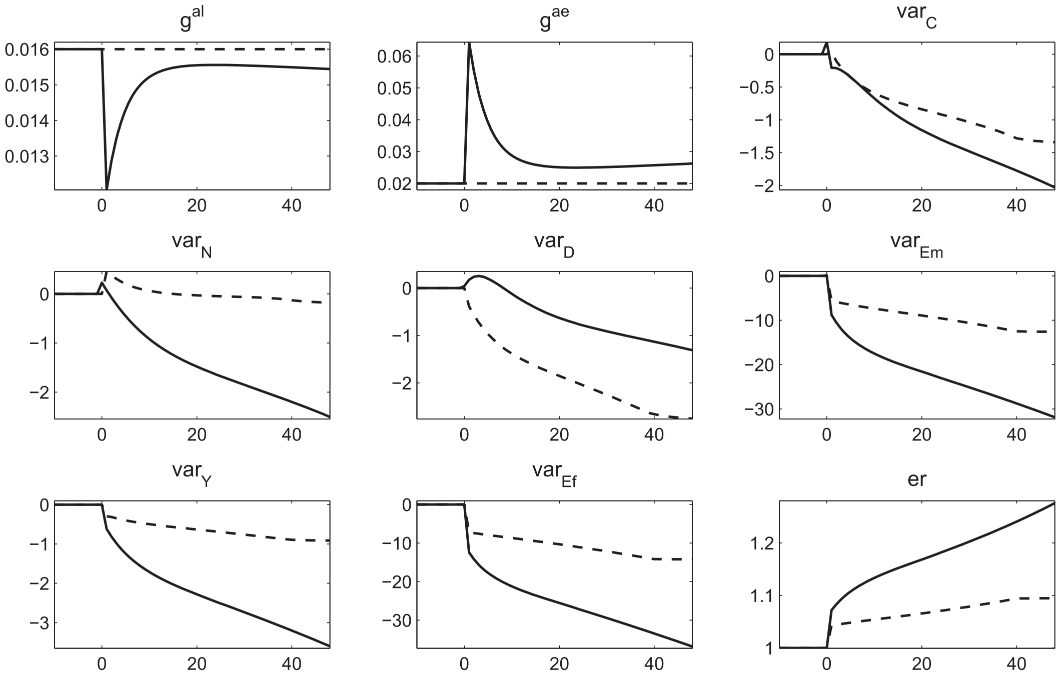
	Simulation 1 $sh = 0.90$	Simulation 1 $sh = 0.90$	Simulation 2 $sh = 0.90$	Simulation 3 $sh = 0.90$	Simulation 4 $sh = 0.90$
E_{2050}/E_{2010}	0.61	0.60	0.59	0.43	0.25
φ (%)	0.63	−1.21	−2.61	−1.59	−3.29

As explained in the previous paragraph, when the initial value of sh is equal to 0.99, we find a welfare gain associated with the reform. When $sh = 0.9$, we find a welfare loss (see the first two columns of Table 6). The decrease in fossil fuel use is almost the same in both cases. We now only present the simulations with $sh = 0.9$, for the reasons explained above. Figure 1 represents a number of economic variables, over time, when the carbon tax in the Quinet report is implemented, with exogenous technical progress (dashed line) and when the direction of technical progress is endogenous (solid line). The variable var_C stands for the percentage change of C, compared to its baseline value.[15] The Quinet tax results in an increase in the rate of energy-saving technical progress,

15. Note that it is not a percentage change compared to a date 0 value, but a percentage change compared to a baseline value at the same date. So that if $var_{Ef} = -0.3$ in 2050 for instance, this means that E_f is equal to 70% of its baseline value in 2050, which is less than 70% of its value in 2010, as the energy use decreases from 2010 to 2050 in the baseline.

from 2% to more than 6% initially. This comes at a small cost in terms of overall growth: the labour-saving technical progress decreases from 1.6% to 1.2%. There is a 40% reduction in energy consumption associated with the reform, whereas there was a 26% reduction in the exogenous model, with the same shock. The decrease in demand for energy is due to the increase in the price and the increase in energy efficiency.

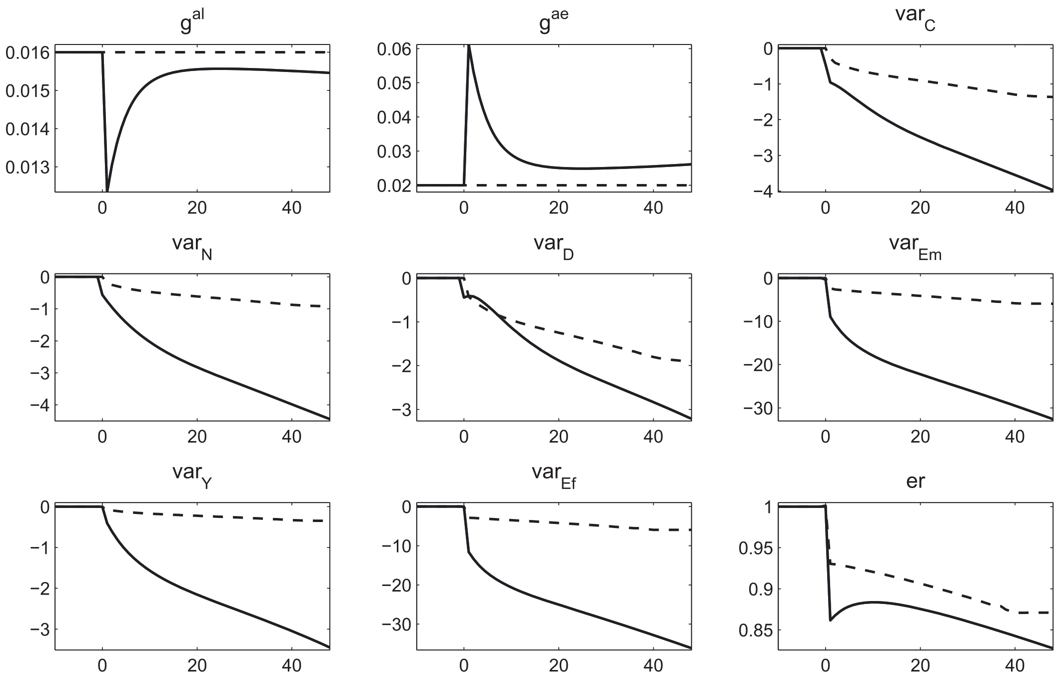
Figure 1: Carbon Tax of the Quinet Report, Exogenous (dashed line) and Endogenous (solid line) TP



3.4.2. Simulation 2: oil shock

We simulate the same oil shock as in the exogenous model. The shock on the oil price is as follows: from date 1 to date 40, the foreign oil price is the sum of the baseline price (increasing at a rate of 2%) and an additional price component increasing at a rate of 4% (like the Quinet tax in Simulation 1). The initial value of this additional price component is equal to 40% of the baseline price at date 1, as in the exogenous model. Figure 2 represents a number of economic variables over time, when the oil shock is simulated, with exogenous technical progress (dashed line) and when the direction of technical progress is endogenous (solid line). The oil shock results in an increase in the rate of energy-saving technical progress, from 2% to 6% initially. This comes at a small cost in terms of overall growth: the labour-saving technical progress decreases from 1.6% to 1.2%. As in the exogenous model, the welfare loss is higher than in Simulation 1, in which the price increase is triggered by a carbon tax (see the third column of Table 6).

Figure 2: Oil Shock, Exogenous (dashed line) and Endogenous (solid line) TP



3.4.3. Simulation 3: subsidy to energy saving R&D

Even when the direction of technical progress is endogenous, the Quinet tax alone is not sufficient to reduce fossil fuel consumption by 75%. We simulate the impact of a Quinet tax associated with a subsidy τ_r of 20% on energy saving R&D, such that the research profit in the energy-saving sector is increased by 20%. The results are presented in Figure 3 (dotted line). The increase in the rate of energy-saving technical progress is huge, as it rises from 2% to almost 20%. The labor-saving technical progress on the other hand decreases to almost zero. The decrease in production is almost twice as great as it was with the Quinet tax alone (solid line in Figure 3). Even with this huge effort toward energy-saving technical progress, energy consumption decreases only by 60% within forty years (see the fourth column of Table 6). Increasing the subsidy even more would redirect all research efforts toward energy-saving technical progress and make the rate of labour-saving technical progress equal to zero for some time, which does not seem realistic or acceptable. In order to achieve a 75% reduction, we run another simulation, in which we increase the initial tax, as well as its rate of growth.

3.4.4. Simulation 4: Factor 4

In this last simulation, we simulate the impact of the following carbon tax value: its initial value is equal to 64 € per ton of CO_2, and it grows at a rate of 8%. In Figure 4, the solid line represents the result of this simulation in the endogenous model, whereas the dashed line represents the results of the same simulation in the exogenous model. In the endogenous model, this carbon tax is enough to achieve the 75% target, but the welfare loss is high (equivalent to a forty years consumption loss of 3.29%, see Table 6). Contrary to previous simulations, the rate of labor-saving technical progress decreases over a long period. As a result, the production *Y* continues to decrease over time compared to its baseline value.

Figure 3: Carbon Tax of the Quinet Report, Endogenous Model, with Subsidy toward Energy Saving Research (dotted line) and without (solid line)

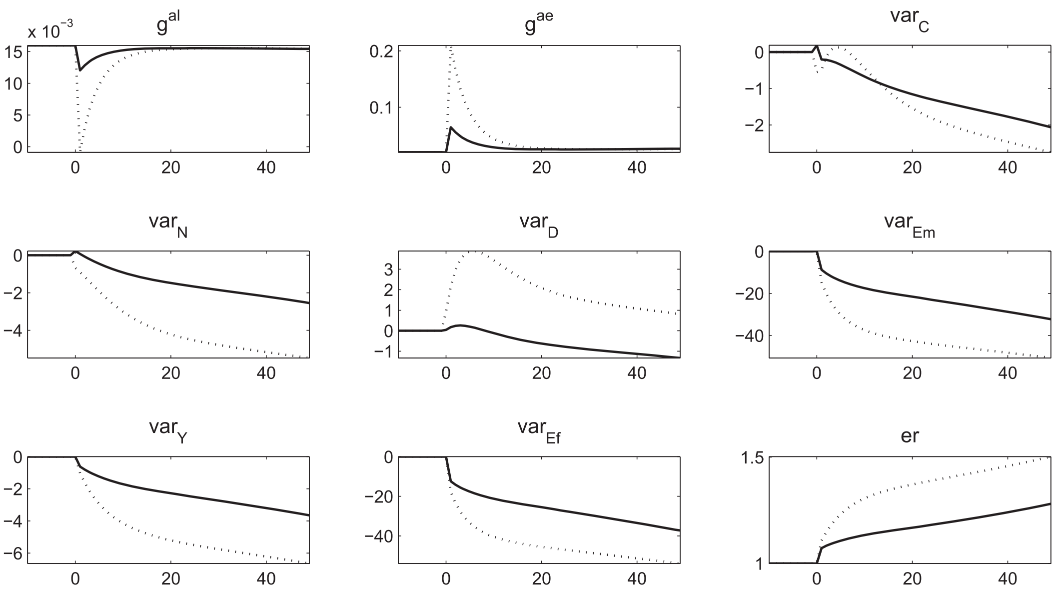
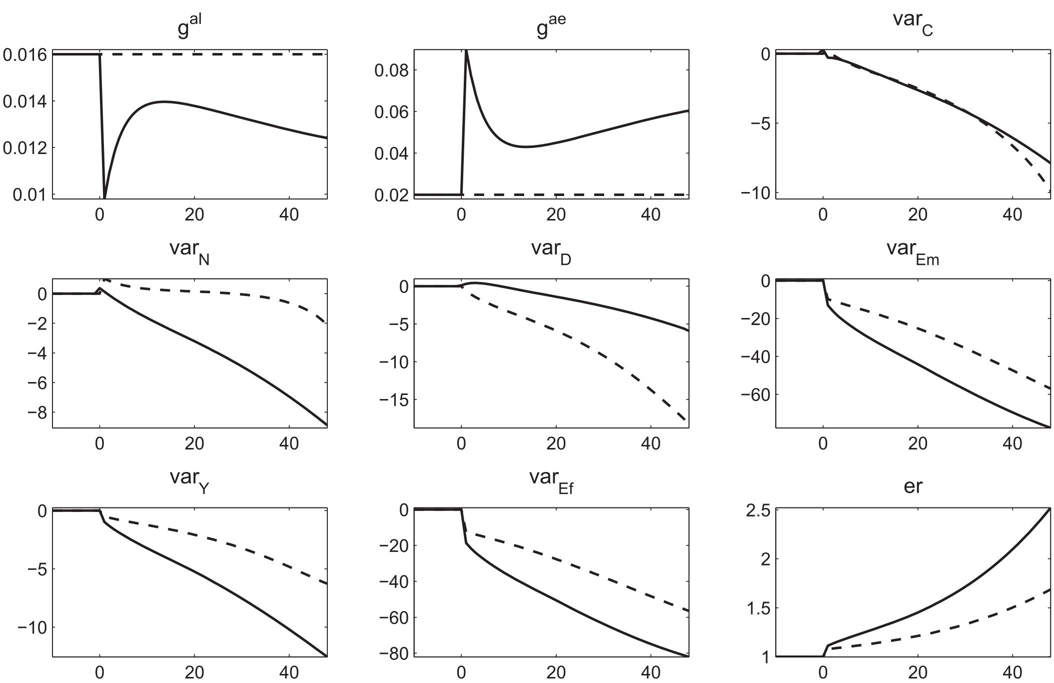


Figure 4: Carbon Tax (initial value 0.3, increasing at 8% rate), Exogenous (dashed line) and Endogenous (solid line) TP



4. CONCLUSION

The conclusion we can draw from our simulations, both in the exogenous version of our model and in its endogenous version, is that the Factor 4 goal is very difficult to achieve for France. Even when we make the rate of energy-saving technical progress endogenous, a 75% reduction in emissions seems almost impossible to achieve. According to our model, it would require a high carbon tax and/or high subsidies to fossil energy-saving technical progress and this would necessarily entail reduced growth for some time.

The results depend of course on the calibration, as do the results from existing large applied models. We have tried to make the model and the calibration as simple and transparent as possible, in order to disentangle the different effects. What seems very robust is the need for fossil energy-saving technical progress to achieve a substantial reduction in emissions. And this technical progress is likely to come at some cost. We wonder if the existing large applied models commonly used to study environmental policy are not misleading in the sense that they under-estimate the magnitude of the effort required to achieve the objective.

We find that achieving a significant reduction in emissions will probably come at a large cost in terms of welfare loss. This result is all the more striking that it relies on optimistic assumptions. In particular, we did not model the fact that increasing the price of energy unilaterally would result in increased imports, loss of competitiveness and carbon leakages. This would worsen our results. This confirms the need for a global environmental policy, at least at the European level. Unfortunately, an ambitious European policy does not seem to be on the agenda, as the financial crisis has overshadowed environmental concerns.

However, our pessimistic results should be put into perspective, as we did not take into account carbon capture and sequestration, which represents an important part of the lowest cost greenhouse gas mitigation portfolio. Taking this technology into account would alleviate the pessimistic conclusion of the article. Moreover, we do not take into account the negative external effect of pollution here. This is not very realistic, but we made this choice in order to focus on a worst-case scenario, and answer this question: what is the loss associated with the reform if it does not yield any positive benefit? We think this question is relevant as the estimates of the benefits from mitigation vary greatly across studies.

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APPENDIX A. ESTIMATION OF THE ELASTICITY OF SUBSTITUTION BETWEEN DURABLE GOODS AND ENERGY AND OF THE RATE OF TECHNICAL PROGRESS

Method

We follow Ogaki & Reinhart (1998) to identify a cointegration relation and estimate the intratemporal elasticity of substitution between E_h and D. Let's denote W_t the intertemporal welfare at date t. With obvious notations, first order conditions with respect to X_t and $E_{h,t}$ lead to:

$$\frac{(1+\tau_t^c)P_t^x}{P_t^e+\tau_{h,t}}=\frac{W_{X_t}}{W_{E_{h,t}}}$$

We have

$$W_{X_t}=\sum_{i=1}^{\infty}\frac{1}{(1+\mu)^{t+i}}\frac{\partial \ln C_{t+i}}{\partial X_t}=\sum_{i=1}^{\infty}\frac{1}{(1+\mu)^{t+i}}\frac{\partial \ln C_{t+i}}{\partial D_{t+i-1}}\frac{\partial D_{t+i-1}}{\partial X_t}$$

From $D_t=\sum_{j=0}^{\infty}(1-\delta_d)^j X_{t-j}$, we deduce $\frac{\partial D_{t+i-1}}{\partial X_t}=(1-\delta)^{i-1}$. Hence

$$W_{X_t}=\sum_{i=1}^{\infty}\frac{1}{(1+\mu)^{t+i}}(1-\delta)^{i-1}\frac{\partial \ln C_{t+i}}{\partial D_{t+i-1}}$$

The expression of $W_{E_{h,t}}$ is straightforward, and we obtain:

$$\frac{(1+\tau_t^c)P_t^x}{P_t^e+\tau_{h,t}}=\frac{\sum_{i=1}^{\infty}\frac{1}{(1+\mu)^{t+i}}(1-\delta)^{i-1}\frac{\partial \ln C_{t+i}}{\partial D_{t+i-1}}}{\frac{\partial \ln C_t}{\partial E_{h,t}}}$$

From F.O.C, we have the following relationship:

$$\frac{\partial \ln C_t}{\partial D_{t-1}}=\frac{v}{1-v}\frac{1}{A_t^e}\left(\frac{D_{t-1}}{A_t^e E_{h,t}}\right)^{-\frac{1}{\varepsilon}}\frac{\partial \ln C_t}{\partial E_{h,t}}$$

so that

$$\frac{(1+\tau_t^c)P_t^x}{P_t^e+\tau_{h,t}}=\sum_{i=1}^{\infty}\frac{1}{(1+\mu)^{t+i}}(1-\delta)^{i-1}\frac{v}{1-v}\frac{1}{A_{t+i}^e}\left(\frac{D_{t+i-1}}{A_{t+i}^e E_{h,t+i}}\right)^{-\frac{1}{\varepsilon}}\frac{\frac{\partial \ln C_{t+i}}{\partial E_{h,t+i}}}{\frac{\partial \ln C_t}{\partial E_{h,t}}}$$

Multiplying both sides by $A_t^e\left(\frac{X_t}{A_t^e E_t}\right)^{\frac{1}{\varepsilon}}$, we get:

$$\frac{(1+\tau_t^c)P_t^x}{(P_t^e+\tau_{h,t})/A_t^e}\left(\frac{X_t}{A_t^e E_t}\right)^{\frac{1}{\varepsilon}}=\sum_{i=1}^{\infty}\frac{1}{(1+\mu)^{t+i}}(1-\delta)^{i-1}\frac{v}{1-v}\frac{A_t^e}{A_{t+i}^e}\left(\frac{D_{t+i-1}}{X_t}\right)^{-\frac{1}{\varepsilon}}\left(\frac{A_t^e E_{h,t}}{A_{t+i}^e E_{h,t+i}}\right)^{-\frac{1}{\varepsilon}}\frac{\frac{\partial \ln C_{t+i}}{\partial E_{h,t+i}}}{\frac{\partial \ln C_t}{\partial E_{h,t}}} \tag{48}$$

If we show that the discounted sum in the right-hand side member of (48) is stationary, we can conclude that the left-hand side member is also stationary. Assuming that A_t^e grows at a constant growth rate, $A_t^e=A_0^e(1+g^{ae})^t$, we will hence be able to derive the following long run relationship:

$$\ln\left(\frac{E_t}{X_t}\right)=\ln\left(\frac{(1+\tau_t^c)P_t^x}{P_t^e+\tau_{h,t}}\right)+(\varepsilon-1)\ln A_0^e+(\varepsilon-1)g^{ae}t+c+u_t \tag{49}$$

where c is a constant and u_t is a white noise. Taking the logarithm of each term of the sum, we see that a necessary condition for stationarity is that $\ln(A_t^e)$, $\ln(X_t)$, $\ln(A_t^e E_{h,t})$ and $\ln\left(\frac{\partial ln\, C_{t+i}}{\partial E_{h,t+i}}\right)$ are difference stationary. Indeed, if a variable V_t is difference stationary, $v_t=\ln(V_t)$ is also difference stationary and $\ln\left(\frac{V_t}{V_{t+i}}\right)=\ln(V_t)-\ln(V_t+i)$ is by definition stationary. This is straightforward for each term, except for the ratio between durable stock and expense. In that last case, we must note that $\frac{D_{t+i-1}}{X_t}$ can be written $\sum_{j=1}^{\infty}(1-\delta_d)^j\frac{X_{t+i-1-j}}{X_t}$, i.e. a sum of stationary terms if X_t is difference stationary.

Our assumption on the linear form of A_t^e implies that A_t^e is difference stationary with drift. Concerning $A^e E_{h,t}$, taking the logarithm shows that it is difference stationary if $E_{h,t}$ is difference stationary. Consequently, we perform below stationarity tests on X_t and $E_{h,t}$. We show that both series are different stationary. The last point concerns the marginal utility of energy consumption $\frac{\partial \ln C_t}{\partial E_{h,t}}$. We cannot prove empirically that its growth rate is stationary, but like Ogaki & Reinhart (1998), we consider that the non-stationarity of this growth rate is unlikely to be empirically important.

We assess the stationarity of $x_t = \ln(X_t)$ and $e_t = \ln(E_{h,t})$ by performing augmented Dickey Fuller tests. We use annual data between 1959 and 2010 for x_t, and between 1973 and 2010 for e_t. We conclude that x_t is difference stationary with a drift and a trend, and that $e_{h,t}$ is difference stationary with a drift, as presented in Table 7.

Table 7: Unit Root Tests

	ADF test		
Variable	stat	$c_{50\%}$	Conclusion
E_h	−3.13	−3.45	I(1) + trend
X	−2.87	−2.93	I(1) + drift

Estimation

Going back to equation (48), we can now draw the following conclusions: 1) $\frac{D_{t+i-1}}{X_t}$ is stationary as the ratio between two difference stationary variables with the same drift; 2) $\frac{A_t^e E_{h,t}}{A_{t+i}^e E_{h,t+i}}$ is stationary as the ratio of difference stationary variables. Consequently, we can now test the cointegration restriction. First, we estimate with OLS the following equation:

$$\ln\left(\frac{E_{h,t}}{X_t}\right) = \alpha + \beta t + \gamma \ln\left(\frac{(1+\tau_t^c)P_t^x}{P_t^e + \tau_{h,t}}\right) + u_t$$

Table 8: Estimation of the Cointegration Relation between $\frac{X}{E_h}$ and $\frac{(1+\tau^c)P^x}{P^e + \tau_h}$

sample period	α	β	γ
1960–2010	15.105	−0.008138	0.495
	(4.7)	(−5.05)	(5.23)

Then we test the stationarity of residuals. We perform a Dickey Fuller test on residuals. We find a statistic $t = -2.37$ which is below the 5% critical value of -1.95. We conclude that we can reject the unit root hypothesis and that the residuals are stationary. Comparing the estimated equation with equation (49), we see that γ is the elasticity of substitution ε and that β is equal to $(\varepsilon - 1)g^{ae}$. So we have the following results: the elasticity of substitution between E_h and D is $\varepsilon = 0.50$ and the rate of technical progress is $g^{ae} = 1.6\%$.

APPENDIX B. ESTIMATION OF THE ELASTICITIES OF SUBSTITUTION AND THE RATES OF TECHNICAL PROGRESS IN PRODUCTION

Method

The two-level production function is given by equations (10) and (11). We deduce from the first order conditions (15)–(17), denoting $\hat{x} = \frac{\dot{X}}{X}$ and omitting the time index:

$$\hat{l} - \hat{y} = (\rho - 1)\hat{a}^l + \rho(\hat{p}^l - \hat{p}^y) \tag{50}$$

$$\hat{z} - \hat{y} = \rho(\hat{p}^y - \hat{p}^z) \tag{51}$$

$$\hat{e} - \hat{z} = (\sigma - 1)\hat{a}^e + \sigma(\hat{p}^z - \hat{p}^e) \tag{52}$$

$$\hat{k} - \hat{z} = \sigma(\hat{p}^z - \hat{p}^k) \tag{53}$$

But $\hat{z}$ and $\hat{p}^z$ cannot be observed. To get rid of them, we first add $\hat{p}^k - \hat{p}^z$ on both sides of (53):

$$\hat{p}^k + \hat{k} - (\hat{p}^z + \hat{z}) = (\sigma - 1)((\hat{p}^z - \hat{p}^y) - (\hat{p}^k - \hat{p}^y))$$

then use (51) to obtain

$$(\rho - 1)(\hat{p}^z - \hat{p}^y) = \hat{p}^y + \hat{y} - (\hat{p}^z + \hat{z})$$

so that:

$$\hat{p}^k + \hat{k} - (\hat{p}^z + \hat{z}) = (\sigma - 1)\left(\frac{\hat{p}^y + \hat{y} - (\hat{p}^z + \hat{z})}{\rho - 1} - (\hat{p}^k - \hat{p}^y)\right)$$

Then, denoting θ_{KZ} the variation of the share in value of K in Z, we get:

$$\theta_{KZ} = \frac{\sigma - 1}{1 - \rho}\theta_{ZY} + (1 - \sigma)(\hat{p}^k - \hat{p}^y)$$

Now adding to both sides of (52) the term $\hat{p}_e - \hat{p}_z$ and using (51), we get:

$$\theta_{EZ} = (\sigma - 1)\hat{a}^e + \frac{\sigma - 1}{1 - \rho}\theta_{ZY} + (1 - \sigma)(\hat{p}^e - \hat{p}^y)$$

Finally, we have the following three equations:

$$\hat{l}-\hat{y}=(\rho-1)\hat{a}^l+\rho(\hat{p}^l-\hat{p}^y) \tag{54}$$

$$\theta_{KZ}=\frac{\sigma-1}{1-\rho}\theta_{ZY}+(1-\sigma)(\hat{p}^k-\hat{p}^y) \tag{55}$$

$$\theta_{EZ}=(\sigma-1)\hat{a}^e+\frac{\sigma-1}{1-\rho}\theta_{ZY}+(1-\sigma)(\hat{p}^e-\hat{p}^y) \tag{56}$$

so that we have to estimate the system:

$$y_1=\alpha_1+\beta_1 x_1+\epsilon_1 \tag{57}$$

$$y_2=\beta_{21}x_{21}+\beta_{22}x_{22}+\epsilon_2 \tag{58}$$

$$y_3=\alpha_3+\beta_{31}x_{31}+\beta_{32}x_{23}+\epsilon_3 \tag{59}$$

We proceed as in van de Werf (2008). We estimate first the first equation separately. Then we estimate the system of the two last equations, with the following restrictions on coefficients: $\beta_{31}=\beta_{21}=-\frac{\beta_{22}}{1-\beta_1}$ and $\beta_{22}=\beta_{32}$.

Results

If we do not include gas in the data (data on gas come from another source, Ceren) and run the regression from 1986, we find similar results: the elasticity of substitution between L and Z_f is $\rho=0.52$, the elasticity of substitution between K and E_f is $\sigma=0.48$, and the energy and labor efficiency growth rates are respectively $g^{ae}=2.4\%$ and $g^{al}=1.5\%$ (significant also). Including gas, we find that the elasticity of substitution between L and Z_f is $\rho=0.52$. The elasticity of substitution between K and E_f is $\sigma=0.52$. The energy efficiency growth rate is $g^{ae}=2.7\%$ and the labour efficiency growth rate is $g^{al}=1.5\%$. All the results are significant at 5% at least.

These results are consistent with those of Lalanne et al. (2009).

APPENDIX C. ENERGY TAXES

This appendix presents what taxes on fossil energy represent in comparison to the initial price without taxes, for households and firms. Fossil fuels in France are taxed at TICPE (former TIPP) for petroleum products, TICGN for gases, and VAT. TICPE and TICGN are excise duties paid on the quantity consumed, and VAT applies on the price including those taxes. So the consumer price of oil products all taxes included is $P_{ATI}=(P+TICPE)(1+VAT)$.

Many exemptions exist for the payment of TICPE for firms, but no one exists for households. On the contrary, the TICGN is not paid by households. Table 9 shows how the price is decomposed for each energy from fossil origin consumed by households. We do not present the details for gas consumption because it is only subject to VAT at normal rate 19.6%.

Table 9: Decomposition of the Price of Energy from Fossil Origin for 2010 in /hl. and Impact of a 32€/tCO_2 Tax

	Price decomposition in €/hl				Emission factor in kg CO_2/ hl	Extra cost of a 32€/tCO_2 tax in €/hl
	Price before taxes	Taxes on energy	VAT	Total price		
Diesel	53.1	42.8	18.8	114.7	268	8.6
Gasoline	52.0	60.6	22.1	134.6	242	7.7
Domestic fuel	54.2	5.7	11.7	71.6	268	8.6
Liquefied gas	55.7	6.0	12.1	73.8	158	5.1

Source: Direction Générale de l'Energie et du Climat (DGEC), Ademe

Table 9 shows that the tax burden largely depends on the type of energy. We compute an average tax rate, including specific taxes on energy and VAT for households and firms. Households' consumption of fossil energies is available with details, as shown in Table 10. With the tax rates presented in Table 9, we find that the average tax rate for households' fossil energy consumption represents 77% of the price before tax (see Table 10).

Table 10: Households' Energy Consumption in Billions of €

	Consumption incl. taxes	Consumption before taxes	Taxes on energy	VAT	Total taxes	As a % of cons. before taxes
Diesel	22.2	10.3	8.3	3.7	11.9	116
Gasoline	13.4	5.2	6.0	2.2	8.2	159
Domestic fuel	7.1	5.4	0.6	1.2	1.7	32
Liquefied gas	1.7	1.3	0.1	0.3	0.4	32
Natural gas	11.0	9.2	0	1.8	1.8	20
Total	55.4	31.3	15.0	9.1	24.1	77

Source: INSEE, DGEC and authors computation

Concerning firms, intermediate consumption only exists at an aggregate level, preventing us from applying the same procedure. We start from total energy taxes on petroleum products, and infer the part paid by firms by subtracting the part paid by households from the total. We then add TIGCN, which is only paid by firms, and compute the average rate of taxes, in regard to firms intermediate consumption of fossil energies. Total TICPE collected in 2010 is € 23.9 bn (source: DGEC), whereas TICGN is € 0.3 bn. From our € 15 bn estimation of TICPE paid by households, we obtain that TICPE paid by firms is € 8.9 bn. So total taxes are € 9.2 bn. Some firms are also covered by the European Trading Scheme, which puts constraint on their emissions. More precisely, the system puts an excess cost to fossil energy (emitting GHG), that can be interpreted as taxation. In 2010, the average price of permits was 13 €, and annual permits which were distributed amounted to 132 millions. We thus consider that it implies an excess cost of € 1.8 bn for firms.

We then retrieve from intermediate consumption of firms the amount paid for energy from fossil origin. This corresponds to € 51.3 bn (tax on energy included) in 2010. So the tax rate applied to firms on energy from fossil origin is $\frac{9.2+1.8}{51.3-9.2}=26\%$.

APPENDIX D. WELFARE LOSS

D.1. Exogenous Model

The intertemporal welfare writes:

$$W_t = \sum_{s=1}^{\infty} \frac{\ln C_{t+s}}{(1+\mu)^s}$$

or:

$$W_t = \frac{1}{1+\mu}(\ln C_{t+1} + W_{t+1})$$

But

$$C_t = A_t^l c_t = A_0^l \prod_{j=0}^{t} (1 + g_j^{al}) c_t$$

where c_t is stationary. So that:

$$W_t = \sum_{s=1}^{\infty} \frac{\ln (A_0^l \prod_{j=0}^{t+s} (1 + g_j^{al}) c_{t+s})}{(1+\mu)^s} = \sum_{s=1}^{\infty} \frac{\ln A_0^l + \sum_{j=0}^{t+s} \ln (1 + g_j^{al}) + \ln c_{t+s}}{(1+\mu)^s}$$

$$= \ln A_0^l \underbrace{\sum_{s=1}^{\infty} \frac{1}{(1+\mu)^s}}_{\frac{1}{\mu}} + \sum_{s=1}^{\infty} \left(\frac{1}{(1+\mu)^s} \sum_{j=0}^{t+s} \ln (1 + g_j^{al}) \right) + \underbrace{\sum_{s=1}^{\infty} \frac{\ln c_{t+s}}{(1+\mu)^s}}_{w_t}$$

and:

$$\frac{1}{\mu} \ln A_0^l + \sum_{s=1}^{\infty} \left(\frac{1}{(1+\mu)^s} \sum_{j=0}^{t+s} \ln (1 + g_j^{al}) \right) + w_t$$

$$= \frac{1}{1+\mu} \ln \left(A_0^l \prod_{j=0}^{t+1} (1 + g_j^{al}) c_{t+1} \right)$$

$$+ \frac{1}{1+\mu} \left(\frac{1}{\mu} \ln A_0^l + \sum_{s=1}^{\infty} \left(\frac{1}{(1+\mu)^s} \sum_{j=0}^{t+1+s} \ln (1 + g_j^{al}) \right) + w_{t+1} \right)$$

i.e.

$$\frac{1}{\mu}\ln A_0^l + \sum_{s=1}^{\infty}\left(\frac{1}{(1+\mu)^s}\sum_{j=0}^{t+s}\ln(1+g_j^{al})\right) + w_t$$

$$= \frac{1}{1+\mu}\left(\ln A_0^l + \sum_{j=0}^{t+1}\ln(1+g_j^{al}) + \ln c_{t+1}\right)$$

$$+ \frac{1}{1+\mu}\left(\frac{1}{\mu}\ln A_0^l + \sum_{s=1}^{\infty}\left(\frac{1}{(1+\mu)^s}\sum_{j=0}^{t+1+s}\ln(1+g_j^{al})\right) + w_{t+1}\right)$$

$$= \frac{1}{\mu}\ln A_0^l + \frac{1}{1+\mu}\sum_{j=0}^{t+1}\ln(1+g_j^{al}) + \sum_{s=1}^{\infty}\left(\frac{1}{(1+\mu)^{s+1}}\sum_{j=0}^{t+1+s}\ln(1+g_j^{al})\right)$$

$$+ \frac{1}{1+\mu}(\ln c_{t+1} + w_{t+1})$$

i.e.

$$\sum_{s=1}^{\infty}\left(\frac{1}{(1+\mu)^s}\sum_{j=0}^{t+s}\ln(1+g_j^{al})\right) + w_t$$

$$= \frac{1}{1+\mu}\sum_{j=0}^{t+1}\ln(1+g_j^{al}) + \sum_{s=2}^{\infty}\left(\frac{1}{(1+\mu)^s}\sum_{j=0}^{t+s}\ln(1+g_j^{al})\right) + \frac{1}{1+\mu}(\ln c_{t+1} + w_{t+1})$$

i.e.:

$$w_t = \frac{1}{1+\mu}(\ln c_{t+1} + w_{t+1})$$

Along the steady state:

$$w = \frac{\ln c}{\mu}$$

Call $w(1)$ the intertemporal welfare in the initial steady state (before the shock) i.e. $w(1) = \ln c/\mu$, and $w(2)$ the intertemporal welfare in the simulation. We seek to estimate the welfare loss during the reform, that is to say during forty years. The welfare in the initial steady state, during forty years, is equal to:

$$\tilde{w}(1) = w(1)\left(1 - \frac{1}{(1+\mu)^{40}}\right)$$

Similarly

$$\tilde{w}(2) = w(2) - \frac{w(42)}{(1+\mu)^{40}}$$

but

$$\tilde{w}(1) = \ln(c)\frac{1-\frac{1}{(1+\mu)^{40}}}{\mu}$$

Similarly

$$\tilde{w}(2) = \ln((1+\varphi)c)\frac{1-\frac{1}{(1+\mu)^{40}}}{\mu}$$

Finally:

$$\varphi = \exp\left((\tilde{w}(2)-\tilde{w}(1))\frac{\mu}{1-\frac{1}{(1+\mu)^{40}}}\right)$$

D2. Endogenous Model

Let C_t be the consumption at date t in the simulation and C_t^{bl} the consumption at date t in the baseline. One can verify that (the shock occurs at date 2 in Dynare):

$$\varphi = \exp\left(\frac{\mu}{1-\frac{1}{(1+\mu)^{40}}}\sum_{2}^{41}\frac{\ln\left(\frac{C_t}{C_t^{bl}}\right)}{(1+\mu)^{t-1}}\right)$$