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# A Stochastic Dominance Analysis of Growth, Recessions and the U.S. Income Distribution, 1967–1986\*

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#### I. Introduction

The adverse effects of recessions and the benefits of growth are widely researched and generally well understood. But the effects of recessions and low growth on the distribution of income have not been so extensively researched nor are they well understood. This paper applies stochastic dominance, a relatively new approach to studying income distributions, to investigate the effects of recent growth and recessions on the U.S. income distribution. Rather than looking at a single summary statistic such as per capita GNP across time, we consider the entire income distribution and measure economic performance by looking at a number of points within the distribution. Our objective is twofold. First, we use recent developments in income distribution theory and measurement to present inference based measures of the U.S. income distributions across time. Second, we address the question of how recent growth and recessions have impacted on the well-being of specific income groups and classes.

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1. A number of writers have emphasized the effects of growth and recessions on poverty. Of course, poverty is related but does not capture the entire income distribution. Several writers beginning in the late 1960s and early 1970s analyzed the effects of recessions on the relative income distribution. The work of Metcalf [11], Thurow [20] and Mirer [12] is particularly noteworthy. However, this early work was contemporaneous with Atkinson's theoretical insight and failed to incorporate important developments in the stochastic dominance measurement of income distributions or statistical inference tests of the sort used in this paper. Later writers, particularly Blinder and Esaki [7], Beach [2] and Blank and Blinder [6] focus on how specific macroeconomic variables (e.g., the unemployment rate), effect particular cardinal measures of relative income distributions across time. For a recent review of this work see the second half of Beach's [3] survey. Like the work in the early 1970s, none of the more recent analysis of relative income distributions uses the stochastic dominance framework or statistical inference procedures relating to changes in income distribution functions.

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Stochastic dominance was developed in financial economics in the 1960s and 1970s to analyze investment portfolios of utility maximizing investors. Increasingly, stochastic dominance is recognized to be a powerful method for analyzing and evaluating income distributions. The original insight dates back to Atkinson [1] who demonstrated that when the means of two distributions are the same, ordinary Lorenz dominance implies stochastic dominance, a condition that carries powerful welfare implications. Extensions of the methodology by Sen [16], Dasgupta, Sen, and Starrett [8], Saposnik [13; 14], Shorrocks [19], and Foster and Shorrocks [10] firmly establish stochastic dominance as a method for partially ordering income distribution functions and ranking levels of economic well-being.

As emphasized by Saposnik, first degree stochastic dominance (FSD) is consistent with the Pareto welfare principle with the important modification of anonymity. The names of income recipients do not matter, only the ordered positions in the distribution are of consequence. For a number of reasons FSD is a logical starting place for applied studies of income distribution. First, distributions ranked in terms of FSD are also ranked by second degree stochastic dominance (so called generalized Lorenz dominance) and all higher degrees of stochastic dominance. Second, compared to other ranking procedures, FSD makes the least restrictive assumptions. Third, other welfare criteria of interest but outside the framework of stochastic dominance can be immediately derived from FSD. Finally, FSD is directly related to a number of commonly used poverty measures. These characteristics give FSD great appeal as a method for analyzing income distributions and evaluating changes in economic well-being.

In this paper we apply standard FSD analysis and a modification of FSD to investigate changes in the U.S. distribution of income over time. The modifications are of two sorts. The first relates to the reluctance of many observers to accept the postulate that an increase in income benefiting only the wealthy, leaving incomes in other positions unchanged, represents an improvement in overall (aggregate) welfare, a proposition quite consistent with the Pareto criterion. To give greater emphasis to the economic well-being of individuals at the bottom of the income distribution a related but more restrictive welfare criterion, which we term conditional FSD, is distinguished. The essence of conditional FSD is that it requires dominance in the lowest decile. The second modification involves truncating the income distributions and applying the FSD principle over segments to assess the effects of year-to-year changes on particular income classes. This permits us to separately evaluate the differential effects of growth and recessions on the economic well-being of individuals in the lower, middle and upper income classes.

The power of FSD in applied studies of income distributions is an empirical question. Using statistical inference procedures to construct partial orders of income distribution functions, Bishop, Formby, and Thistle [5] provide evidence that FSD has surprising power in cross sectional comparisons of income distributions. While detailed results must necessarily be stated in the body of the paper, we find essentially the same results across time; the analysis provides new evidence on the distributional impacts of growth and recessions on the economic status of specific income groups, including the much discussed middle class.

## II. First Degree Stochastic Dominance and Welfare

The relationship between the distribution of income and standards of living, we assume, are summarized in a social welfare or social evaluation function representing the ethical judgments regarding income distributions. To compare income distributions, we employ first degree stochastic

dominance [16; 13; 14], since it embodies the fewest restrictive ethical axioms. First degree sto-chastic dominance (FSD) is based on the strong Pareto principle and anonymity, two assumptions that should have a wide degree of acceptance. To compare populations of different sizes, we adopt the population principle [8; 9; 17]. The population principle states that duplicating the population, including their incomes, exactly doubles the level of welfare, i.e.,  $w_{2n}(x,x) = 2w_n(x)$ . As Sen [18] points out, this implies that the comparisons of income distributions should be interpreted as comparisons of standards of living or per capita welfare.

An important consequence of these assumptions is that together they imply the statistical cumulative distribution function (cdf) for income contains sufficient information for a social evaluation. Formally, let  $F^1$  denote the income cdf for period 1. It is more convenient to use the inverse distribution function  $X^1(p) := \inf\{x : F^1(x) \ge p\}$ ,  $p \in [0, 1]$ . The inverse distribution or quantile function can be thought of as giving individual's incomes in increasing order. Then period 1 dominates period 2 iff  $X^1(p) \ge X^2(p)$  for all  $p \in [0, 1]$ . If  $X^1(p) = X^2(p)$  for all  $p \in [0, 1]$ , then the same income distribution and the same standard of living prevails in both periods. If  $X^1(p) > X^2(p)$  for some p, and  $X^1(p) < X^2(p)$  for some p (i.e., the quantile functions cross), the distributions are noncomparable, and the question of which period has the higher standard of living cannot be determined without imposing additional ethical judgments on the welfare function.

To illustrate comparisons of two quantile functions Figure 1 plots the conditional U.S. mean incomes (corrected for price changes) at deciles for 1967 and 1986. The data are taken from Table I, which is discussed in detail below. For present purposes it is sufficient to emphasize that FSD involves comparisons of the absolute incomes of anonymous classes of income recipients at specific points in the income distribution. If the 1986 quantile function is above 1967 at one or more points and not below it at any other point, then, as in Figure 1, 1986 dominates 1967 in terms of FSD. On the other hand, if 1986 were above 1967 at some points and below it at others, the quantile functions would intersect and the two years could not be ranked using FSD. The dominance depicted in Figure 1 permits us to conclude unambiguously that overall economic well-being as measured by FSD was at a higher level in 1986 than in 1967.

First degree dominance, like the Pareto principle upon which it is based, is satisfied across time if the incomes of only the very rich in society rise, so long as other incomes do not fall. The proposition that economic prosperity benefiting only upper income recipients represents an improvement in overall economic well-being of society is frequently questioned and, therefore, often regarded as controversial, i.e., involves value judgments with which many people do not agree. To incorporate a social preference for economic improvements at the bottom of the income distribution we add a further restriction on the welfare function and introduce conditional FSD. According to this welfare criterion, welfare gains by the lowest income group are necessary but not sufficient for an overall improvement in economic well-being. Conditional FSD occurs when the incomes of the poorest members of society rise and other incomes do not fall. Thus, conditional FSD imposes Rawls' maximin principle by requiring dominance in the lowest quantile. Our work

<sup>2.</sup> While the anonymity assumption would be inappropriate in a study of income dynamics and mobility, it proves most useful in evaluating the performance of an economic system in generating incomes in specific positions in the ordered distribution function at different points in time. Population invariance is a non-stringent assumption when making comparisons over short time intervals and is particularly appropriate for the year-to-year comparisons carried out in this paper.

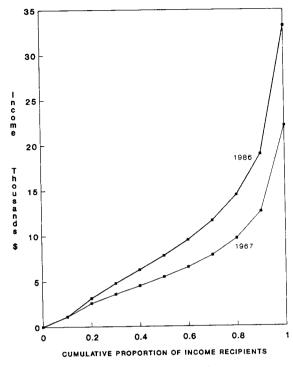


Figure 1. U.S. Income Distributions (Quantile Functions), 1967 and 1986

focuses on deciles, hence, conditional FSD is Pareto dominance cum dominance in the lowest decile.<sup>3</sup> Conditional FSD necessarily implies Paretian FSD, but, the converse does not hold.

FSD and conditional FSD compare entire income distribution functions. Alternative restrictions can be imposed that allow us to evaluate segments of the distribution and analyze the economic well-being of specific income groups including the lower, middle, and upper income classes. Foster and Shorrocks [10] were the first to use first degree dominance in this fashion and demonstrate that dominance over a segment of the income distribution function (those incomes below some arbitrary poverty cutoff) is sufficient for any headcount poverty ordering. In the same manner, truncating the income distribution at both ends and focusing on the middle permits application of the powerful FSD theorem to assess the changes in the economic status of the middle class. Similarly, truncating the distribution at some predetermined high value can be used to evaluate the economic status of upper income recipients.

# III. First Degree Stochastic Dominance and the U.S. Income Distribution—1967 to 1986

We use the Current Population Survey microdata (March tapes) to investigate the sample income distribution functions by applying the welfare theoretic and statistical inference procedures dis-

<sup>3.</sup> In a manner similar to our concept of conditional FSD Saposnik and Tutterow [15] combine the Rawlsian maximin principle with FSD to evaluate the welfare consequences of income dynamics and mobility.

cussed above. We analyze and compare the entire income distribution functions and segments of the distributions on a year-by-year basis for the period 1967 to 1986.

The income concept used is Census total money income and the income recipient unit is the individual family member.<sup>4</sup> For example, a family of four with \$40,000 in Census total money income is reported as four incomes of \$10,000 each. Incomes are reported in 1986 dollars using the GNP personal consumption expenditures deflator.

#### Statistical Inference

To make FSD comparisons, we estimate the income distribution (quantile) functions as vectors of decile conditional means. We define  $\mu_i$  as the *i*th conditional mean (i = 1, 2, ..., k). Making use of important contributions by Beach and Davidson [4], Bishop, Formby, and Thistle [5] demonstrate that under reasonable, regularity conditions,  $N^{1/2}(\hat{\mu} - \mu)$  has a limiting normal distribution, and provide an expression for the asymptotic variance,  $Q_i$ .

To make inferences, we use the information from the k pairwise tests of the sample conditional means:

$$T_i = (\mu_i^a - \mu_i^b)/[Q_i^a/N^a + Q_i^b/N^b]^{1/2},$$

 $i=1,2,\ldots,k$ . To maintain the size of the joint test of two vectors of sample conditional means, the critical values are determined from the Student Maximum Modulus (SMM) distribution. That is, an approximately  $\alpha$  level test of the equality of two vectors of conditional means rejects each of the k subhypotheses if  $T_i > m_{\alpha}(k, 1)$ , where  $m_{\alpha}(k, 1)$  is the upper  $\alpha$  critical value of the SMM distribution with  $\infty$  degrees of freedom.

The FSD partial orders are determined as follows. If we accept the null hypothesis of equality at all of the conditional means, then we rank the two income distributions as equal. The failure to accept  $H_0$  for any pairwise comparison of conditional means requires us to differentiate between first degree dominance and noncomparability. If  $\mu_i^a > \mu_i^b$  for each pairwise comparison, then a dominance relation results. If  $\mu_i^a > \mu_i^b$  for some pair of conditional means, and  $\mu_i^a < \mu_i^b$  for some other pair of conditional means, then the two income distribution functions are noncomparable.<sup>5</sup>

### Paretian and Conditional FSD Dominance across Time

Table I presents annual decile means, standard errors (in parenthesis), test statistics for the annual comparisons of decile means, welfare rankings, and sample sizes.<sup>6</sup> A welfare ranking of "P"

- 4. Census income, of course, has a number of well known limitations. But, we agree with Sen who addressed the shortcoming of cash income as a measure of well-being and concluded that we cannot "construct from it a picture of total disaster" [17, 78]. There are two related reasons for using census data. First, the statistical procedures used to test for dominance of income distributions require microdata. Second, we seek to test for a large number of year-to-year changes in the distributions and this requires comparable microdata across a long time period. Census microdata satisfies both requirements.
- 5. To maintain the size of the joint test of two vectors of sample conditional means, the critical values are determined from the Student Maximim Modulus (SMM) distribution. At the five percent level of significance the critical SMM value is determined by the number of tests performed in a particular comparison. The critical values are k = 1, SMM = 1.96; k = 2, SMM = 2.24; k = 3, SMM = 2.39; k = 4, SMM = 2.51; k = 10, SMM = 2.80. We use different SMM values depending on the number of deciles being compared.
- 6. While our interest is in comparing annual changes in the (inverse) income distribution functions, we provide the standard errors in Table 1 to allow comparisons of any two time periods.

indicates Paretian dominance, while "C" denotes conditional FSD. "Cross" indicates a statistically significant intersection of the income distribution (quantile) functions. As explained above, a crossing implies that the two distributions are noncomparable using the FSD criteria. A "+" denotes the dominance of the later year over the earlier year, while a "-" indicates the dominance of the earlier year over the later. To give emphasis to the fact that conditional FSD implies Paretian dominance each ranking of "C" is accompanied by a "P", denoted as "C,P". However, a ranking of "P" is not necessarily accompanied by a ranking of "C".

To interpret Table I we discuss four specific year-to-year comparisons. First, the mean income of the poorest decile in 1967 is \$1100 (row 1, column 1) with a standard error of \$18, while in 1968 (row 2, column 1) it is \$1229 with a standard error of \$19. The resulting statistic for testing for differences between the first decile mean income in 1967 and 1968 is 4.96. Thus, the mean income of the poorest ten percent of income recipients significantly increased between 1967 and 1968. A comparison of the differences in all the decile means between 1967 and 1968 shows that each decile test statistic exceeds the five percent SMM critical value. Thus the welfare ranking is "C+,P+" denoting low income dominance as well as Paretian FSD of 1968 over 1967.

In contrast to the 1967–68 comparison, tests for differences in the decile means between 1969 and 1970 result in a significant crossing of the quantile functions. The 1969–70 recession reduced income in the lowest decile in 1970, while the ninth decile income increased (all other deciles show no significant change). Intersecting quantile functions mean that individuals in some of the ordered positions in the income distribution suffered welfare losses while others experienced welfare gains. To indicate the noncomparability of the 1969 and 1970 distributions "Cross" is entered in column 2 of Table I. As emphasized above, when a statistically significant crossing occurs it is not possible to order the distributions using the FSD principle.

Now consider 1975 compared to 1976. In this comparison the bottom decile has a statistically insignificant change in conditional mean income, while the second through tenth deciles all experienced significant gains (the top decile being unchanged). The Paretian FSD criterion is satisfied but the requirement for conditional FSD is not. In this case we assign a ranking of "P+". The final comparison we discuss is 1973 versus 1974. The first, fifth, ninth, and tenth deciles all experienced decreases in income while the changes in other deciles are insignificant. In this case there is a conditional as well as a Paretian decline in welfare and the ranking is "C-,P-".

A count of the rankings in Column 2 of Table I reveals that in 17 out of the 19 overall comparisons welfare can be evaluated using the Paretian criterion. Economic well-being improved in 13 of the 19 year-by-year comparisons and declined in four. Conditional dominance occurs in ten of the 19 comparisons, with six of the ten being improvements. It is noteworthy that all declines in overall economic well-being which are indicated by Paretian dominance (P-) are accompanied by conditional dominance (C-), while only about one half of the Paretian improvements in welfare (P+) are characterized by conditional dominance (C+).

It is clear from Table I that recessions have significant effects on the distribution of income with individuals at the low end of the distribution bearing the brunt of the welfare loss. Four recession related year-to-year comparisons are characterized by declining incomes and conditional dominance (C-). The 1973-74, 1974-75, 1979-80 and 1981-82 comparisons all involve significant losses for the lowest income group, while other deciles had either losses or statistically insignificant gains. Accordingly, these year-to-year comparisons are ranked as C-, which implies P-, hence the overall ranking "C-,P-". Interestingly, no recession related year-to-year change involves losses to all deciles; 1974-75 comes close, missing narrowly in the top decile. In addition, two other recession related years, 1969-70 and 1980-81 are characterized by significant

**Table I.** Paretian and Conditional First Degree Stochastic Dominance Comparisons of U.S. Income Distributions, 1967–1986\*

1967		Dominance**	Decile Incomes									
1968   C+,P+   1229   2800   3862   4808   5795   6932   8296   10089   13175   232   1269	Year		1	2	3	4	5	6	7	8	9	10
1968   C+,P+   1229   2800   3862   4808   5795   6932   8296   10089   13175   23	1967	-		2587	3587	4500	5456	6510	7836	9678	12612	22120
(19) (18) (18) (18) (19) (23) (26) (32) (43) (58) (58) (19) (49) (48) (496 8.86 10.77 11.64 10.53 11.82 10.24 8.70 6.65 40 (19) (18) (29) (22) (24) (28) (34) (44) (65) (19) (18) (20) (22) (24) (28) (34) (44) (65) (19) (18) (20) (22) (24) (28) (34) (44) (65) (19) (18) (20) (22) (24) (29) (34) (46) (66) (19) (19) (19) (19) (21) (24) (29) (34) (46) (46) (66) (19) (19) (19) (19) (21) (24) (29) (34) (46) (46) (66) (19) (19) (20) (23) (24) (29) (36) (47) (67) (67) (20) (19) (20) (23) (24) (29) (36) (47) (67) (67) (20) (19) (20) (23) (24) (29) (36) (47) (67) (67) (20) (19) (20) (23) (24) (29) (36) (47) (67) (67) (20) (19) (20) (23) (24) (29) (36) (47) (67) (67) (20) (19) (20) (22) (25) (27) (29) (38) (48) (72) (20) (21) (20) (22) (25) (27) (29) (38) (48) (72) (20) (21) (20) (22) (25) (27) (29) (38) (48) (72) (20) (21) (20) (22) (25) (27) (29) (38) (48) (72) (20) (21) (22) (22) (24) (27) (31) (38) (49) (74) (20) (21) (22) (22) (24) (27) (31) (38) (49) (74) (20) (21) (22) (22) (24) (27) (31) (38) (49) (74) (20) (21) (22) (23) (23) (26) (32) (37) (49) (72) (10) (20) (22) (23) (23) (26) (32) (37) (49) (72) (10) (20) (22) (23) (23) (25) (29) (36) (47) (68) (19) (19) (21) (22) (23) (25) (29) (36) (47) (68) (19) (19) (21) (22) (22) (24) (27) (29) (38) (36) (47) (68) (19) (19) (21) (22) (22) (24) (29) (39) (36) (47) (68) (19) (19) (21) (22) (22) (23) (23) (25) (29) (36) (47) (68) (19) (19) (21) (22) (23) (23) (25) (29) (36) (47) (68) (19) (19) (21) (22) (23) (23) (23) (23) (23) (23) (23			(18)	(16)	(18)	(19)	(23)					(175)
(19) (118) (118) (119) (23) (26) (32) (43) (58) (10 4.96 8.86 10.77 11.64 10.53 11.82 10.24 8.70 6.65 (19) (118) (20) (22) (24) (28) (34) (44) (65) (10 3.23 4.54 6.00 7.99 8.72 8.30 8.76 7.68 6.81 4 (20) (19) (19) (21) (24) (29) (34) (45) (66) (10 -3.63 -0.98 0.01 0.78 0.85 0.99 1.12 1.66 2.83 1 1971 P+ 1261 2923 4097 5158 6226 7420 8912 10965 14253 25 (20) (19) (20) (23) (24) (29) (36) (47) (67) (67) (60 1.65 1.24 2.64 3.02 3.46 3.23 3.04 3.15 2.35 1 1972 C+,P+ 1353 3112 4378 5516 6669 7939 9501 11635 15080 26 (21) (20) (22) (22) (25) (27) (29) (38) (48) (72) (20) (21) (22) (22) (25) (27) (29) (38) (48) (72) (20) (21) (22) (22) (24) (27) (31) (38) (49) (74) (65) (14) (27) (28) (29) (23) (26) (27) (29) (38) (48) (72) (29) (29) (29) (29) (29) (20) (20) (20) (20) (20) (20) (20) (20	1968	C+,P+	1229	2800	3862	4808	5795	6932	8296	10089	13175	23205
1969   C+,P+				(18)		(19)	(23)					(182)
(19) (18) (20) (22) (24) (28) (34) (44) (65) (19) (19) (19) (21) (22) (24) (28) (34) (44) (65) (19) (19) (19) (19) (19) (21) (24) (29) (34) (45) (66) (28) (20) (19) (19) (19) (21) (24) (29) (34) (45) (66) (2.83 in			4.96	8.86	10.77	11.64	10.53	11.82	10.24	8.70		4.30
3.23   4.54   6.00   7.99   8.72   8.30   8.76   7.68   6.81   7.68   6.81   7.68   6.81   7.68   6.81   7.68   6.81   7.68   6.81   7.68   6.81   7.68   6.81   7.68   6.81   7.68   6.81   7.68	1969	C+,P+							8707	10658	13768	24330
1970 Cross												(198)
(20) (19) (19) (21) (24) (29) (34) (45) (66) (67) (27) (36) (37) (48) (48) (66) (47) (67) (28) (48) (48) (48) (48) (48) (48) (48) (4		_			6.00	7.99	8.72	8.30	8.76	7.68	6.81	4.18
1971   P+   1261   2923   4097   5158   6226   7420   8912   10965   14253   25   (20)   (19)   (20)   (23)   (24)   (29)   (36)   (47)   (67)   (67)   (67)   (76)   (7	1970	Cross							8761	10761	14031	24811
1971 P+												(199)
(20) (19) (20) (23) (24) (29) (36) (47) (67) (67) (7) (165) (1.65) 1.24 2.64 3.02 3.46 3.23 3.04 3.15 2.35 1 1972 C+,P+ 1353 3112 4378 5516 6669 7939 9501 11635 15080 26 (21) (20) (22) (25) (27) (29) (38) (48) (72) (20) (21) (20) (22) (25) (27) (29) (38) (48) (72) (20) (21) (20) (22) (25) (27) (29) (38) (48) (72) (20) (21) (22) (22) (24) (27) (31) (38) (49) (74) (20) (21) (22) (22) (24) (27) (31) (38) (49) (74) (20) (21) (22) (22) (24) (27) (31) (38) (49) (74) (20) (21) (22) (22) (24) (27) (31) (38) (49) (74) (20) (21) (22) (23) (23) (23) (26) (32) (37) (49) (72) (10) (21) (22) (23) (23) (23) (26) (32) (37) (49) (72) (10) (21) (22) (23) (23) (25) (29) (36) (32) (37) (49) (72) (10) (20) (22) (23) (25) (29) (36) (47) (68) (10) (20) (22) (23) (25) (29) (36) (47) (68) (10) (20) (22) (23) (25) (29) (36) (47) (68) (10) (20) (22) (23) (25) (29) (36) (47) (68) (10) (20) (22) (23) (25) (29) (36) (47) (68) (10) (20) (22) (23) (25) (29) (36) (47) (68) (10) (20) (22) (23) (25) (29) (36) (47) (68) (10) (20) (22) (23) (25) (29) (36) (32) (37) (49) (36) (47) (68) (10) (49) (49) (49) (49) (49) (49) (49) (49		_					0.85	0.99	1.12	1.66	2.83	1.72
1.65 1.24 2.64 3.02 3.46 3.23 3.04 3.15 2.35 1 1972 C+,P+	1971	P+								10965	14253	25311
1972 C+,P+  1353 3112 4378 5516 6669 7939 9501 11635 15080 26  (21) (20) (22) (25) (27) (29) (38) (48) (72) (2  3.18 6.72 9.34 10.69 12.30 12.72 11.23 9.94 8.41 4  1973 C+,P+  1454 3286 4585 5732 6903 8201 9771 11942 15437 27  (21) (22) (22) (22) (24) (27) (31) (38) (49) (74) (2  3.41 5.72 6.55 6.24 6.12 6.18 5.03 4.45 3.46 1  1974 C-,P-  1336 3261 4543 5645 6776 8082 9664 11752 15122 25  (23) (22) (23) (23) (23) (26) (32) (37) (49) (72) (1  -3.84 -0.78 -1.31 -2.59 -3.41 -2.69 -2.02 -2.75 -3.06 -4  1975 C-,P-  1247 2980 4248 5412 6548 7794 9317 11450 14815 25  (21) (20) (22) (23) (23) (25) (29) (36) (47) (68) (1  -2.83 -9.52 -9.21 -7.11 -6.34 -6.68 -6.67 -4.46 -3.12 -2  1976 P+  1306 3090 4357 5527 6715 8046 9652 11811 15263 60  (19) (19) (21) (22) (22) (24) (29) (34) (44) (63) (1  2.04 4.00 3.59 3.62 4.76 6.08 6.71 5.58 4.85 3  1977 P+  1280 3199 4504 5692 6908 8280 9969 12202 15716 26  (22) (19) (21) (22) (25) (30) (36) (46) (64) (1  -0.89 4.04 4.94 5.33 5.51 5.62 6.38 6.12 5.04 2  1978 C+,P+  1414 3331 4688 5931 7175 8603 10261 12498 16015 26  (23) (24) (26) (27) (32) (35) (41) (54) (75) (1  4.22 4.26 5.53 6.92 6.55 7.06 5.32 4.17 3.03 (2  1979 P+  1357 3327 4739 6018 7331 8775 10509 12875 16534 27  (18) (20) (20) (22) (22) (25) (28) (35) (44) (60) (1  -1.96 -0.14 1.56 2.53 3.87 3.84 4.59 5.38 5.38 3  1980 C-,P-  120 3155 4542 5823 7108 8572 10365 12706 16239 27  (18) (19) (20) (22) (22) (25) (28) (35) (44) (60) (1  -5.40 -6.29 -6.81 -6.28 -6.38 -5.11 -2.29 -2.76 -3.49 -2  1981 Cross  1066 3007 4399 5701 7007 8507 10307 12712 16486 28  (19) (20) (21) (23) (23) (26) (31) (37) (47) (66) (1										(47)	(67)	(203)
(21) (20) (22) (25) (27) (29) (38) (48) (72) (23) (3.18 6.72 9.34 10.69 12.30 12.72 11.23 9.94 8.41 24 1454 3286 4585 5732 6903 8201 9771 11942 15437 27 (21) (22) (22) (22) (24) (27) (31) (38) (49) (74) (27) (3.41 5.72 6.55 6.24 6.12 6.18 5.03 4.45 3.46 1 1752 15122 25 (23) (22) (23) (23) (26) (32) (37) (49) (72) (10 23) (22) (23) (23) (26) (32) (37) (49) (72) (10 23) (22) (23) (23) (26) (32) (37) (49) (72) (10 23) (20) (22) (23) (23) (25) (29) (36) (47) (68) (10 20) (21) (20) (22) (23) (25) (29) (36) (47) (68) (10 20) (19 (19) (19) (21) (22) (24) (29) (34) (44) (63) (10 20) (20) (21) (22) (24) (29) (34) (44) (63) (10 20) (20) (21) (22) (24) (29) (34) (44) (63) (10 20) (20) (21) (22) (25) (30) (36) (47) (68) (47) (68) (10 20) (20) (21) (22) (25) (30) (36) (47) (68) (10 20) (20) (21) (22) (25) (30) (36) (46) (46) (40 20) (20) (21) (22) (25) (30) (36) (46) (46) (40 20) (40) (40) (40) (40) (40) (40) (40) (4		_			2.64	3.02	3.46	3.23	3.04	3.15	2.35	1.76
3.18 6.72 9.34 10.69 12.30 12.72 11.23 9.94 8.41 4 1973 C+,P+	1972	C+,P+							9501	11635	15080	26667
1973 C+,P+  1454 3286 4585 5732 6903 8201 9771 11942 15437 27  (21) (22) (22) (24) (27) (31) (38) (49) (74) (2  3.41 5.72 6.55 6.24 6.12 6.18 5.03 4.45 3.46 1  1974 C-,P-  1336 3261 4543 5645 6776 8082 9664 11752 15122 25  (23) (22) (23) (23) (26) (32) (37) (49) (72) (1  -3.84 -0.78 -1.31 -2.59 -3.41 -2.69 -2.02 -2.75 -3.06 -4  1975 C-,P-  1247 2980 4248 5412 6548 7794 9317 11450 14815 25  (21) (20) (22) (23) (25) (29) (36) (47) (68) (1  -2.83 -9.52 -9.21 -7.11 -6.34 -6.68 -6.67 -4.46 -3.12 -2  1976 P+  1306 3090 4357 5527 6715 8046 9652 11811 15263 26  (19) (19) (21) (22) (24) (29) (34) (44) (63) (1  2.04 4.00 3.59 3.62 4.76 6.08 6.71 5.58 4.85 3  1977 P+  1280 3199 4504 5692 6908 8280 9969 12202 15716 26  (22) (19) (21) (22) (25) (30) (36) (46) (64) (1  -0.89 4.04 4.94 5.33 5.51 5.62 6.38 6.12 5.04 2  1978 C+,P+  1414 3331 4688 5931 7175 8603 10261 12498 16015 26  (23) (24) (26) (27) (32) (35) (41) (54) (75) (1  4.22 4.26 5.53 6.92 6.55 7.06 5.32 4.17 3.03 (0  1979 P+  1357 3327 4739 6018 7331 8775 10509 12875 16534 27  (18) (20) (20) (22) (25) (28) (35) (44) (60) (1  -1.96 -0.14 1.56 2.53 3.87 3.84 4.59 5.38 5.38 3  1980 C-,P-  120 3155 4542 5823 7108 8572 10365 12706 16239 27  (18) (19) (20) (22) (25) (28) (35) (42) (59) (1  -5.40 -6.29 -6.81 -6.28 -6.38 -5.11 -2.92 -2.76 -3.49 -2  1981 Cross 1066 3007 4399 5701 7007 8507 10307 12712 16486 28  (19) (20) (21) (23) (26) (31) (37) (47) (66) (1										(48)	(72)	(212)
(21) (22) (22) (24) (27) (31) (38) (49) (74) (27) (31) (38) (49) (74) (27) (31) (38) (49) (74) (27) (31) (38) (49) (74) (27) (31) (38) (49) (74) (27) (31) (38) (49) (74) (27) (31) (38) (49) (74) (27) (31) (38) (49) (74) (27) (31) (38) (49) (74) (28) (38) (38) (38) (38) (38) (38) (38) (3					9.34	10.69	12.30	12.72	11.23	9.94	8.41	4.62
3.41 5.72 6.55 6.24 6.12 6.18 5.03 4.45 3.46 1  1974 C-,P-  1336 3261 4543 5645 6776 8082 9664 11752 15122 25  (23) (22) (23) (23) (26) (32) (37) (49) (72) (1  -3.84 -0.78 -1.31 -2.59 -3.41 -2.69 -2.02 -2.75 -3.06 -4  1975 C-,P-  1247 2980 4248 5412 6548 7794 9317 11450 14815 25  (21) (20) (22) (23) (25) (29) (36) (47) (68) (1  -2.83 -9.52 -9.21 -7.11 -6.34 -6.68 -6.67 -4.46 -3.12 -2  1976 P+  1306 3090 4357 5527 6715 8046 9652 11811 15263 26  (19) (19) (21) (22) (24) (29) (34) (44) (63) (1  2.04 4.00 3.59 3.62 4.76 6.08 6.71 5.58 4.85 3  1977 P+  1280 3199 4504 5692 6908 8280 9969 12202 15716 26  (22) (19) (21) (22) (25) (30) (36) (46) (64) (1  -0.89 4.04 4.94 5.33 5.51 5.62 6.38 6.12 5.04 2  1978 C+,P+  1414 3331 4688 5931 7175 8603 10261 12498 16015 26  (23) (24) (26) (27) (32) (35) (41) (54) (75) (1  4.22 4.26 5.53 6.92 6.55 7.06 5.32 4.17 3.03 (0  1979 P+  1357 3327 4739 6018 7331 8775 10509 12875 16534 27  (18) (20) (20) (22) (25) (28) (35) (44) (60) (1  -1.96 -0.14 1.56 2.53 3.87 3.84 4.59 5.38 5.38 3  1980 C-,P-  1220 3155 4542 5823 7108 8572 10365 12706 16239 27  (18) (19) (20) (22) (25) (28) (35) (42) (59) (1  -5.40 -6.29 -6.81 -6.28 -6.38 -5.11 -2.92 -2.76 -3.49 -2  1981 Cross  1066 3007 4399 5701 7007 8507 10307 12712 16486 28  (19) (20) (21) (23) (26) (31) (37) (47) (66) (1	1973	C+,P+						8201	9771	11942	15437	27180
1974 C-,P-  1336 3261 4543 5645 6776 8082 9664 11752 15122 25  (23) (22) (23) (23) (26) (32) (37) (49) (72) (1  -3.84 -0.78 -1.31 -2.59 -3.41 -2.69 -2.02 -2.75 -3.06 -4  1975 C-,P-  1247 2980 4248 5412 6548 7794 9317 11450 14815 25  (21) (20) (22) (23) (25) (29) (36) (47) (68) (1  -2.83 -9.52 -9.21 -7.11 -6.34 -6.68 -6.67 -4.46 -3.12 -2  1976 P+  1306 3090 4357 5527 6715 8046 9652 11811 15263 26  (19) (19) (21) (22) (24) (29) (34) (44) (63) (1  2.04 4.00 3.59 3.62 4.76 6.08 6.71 5.58 4.85 3  1977 P+  1280 3199 4504 5692 6908 8280 9969 12202 15716 26  (22) (19) (21) (22) (25) (30) (36) (46) (64) (1  -0.89 4.04 4.94 5.33 5.51 5.62 6.38 6.12 5.04 2  1978 C+,P+  1414 3331 4688 5931 7175 8603 10261 12498 16015 26  (23) (24) (26) (27) (32) (35) (41) (54) (75) (1  4.22 4.26 5.53 6.92 6.55 7.06 5.32 4.17 3.03 0  1979 P+  1357 3327 4739 6018 7331 8775 10509 12875 16534 27  (18) (20) (20) (22) (25) (28) (35) (44) (60) (1  -1.96 -0.14 1.56 2.53 3.87 3.84 4.59 5.38 5.38 3  1980 C-,P-  1220 3155 4542 5823 7108 8572 10365 12706 16239 27  (18) (19) (20) (22) (25) (28) (35) (42) (59) (1  -5.40 -6.29 -6.81 -6.28 -6.38 -5.11 -2.92 -2.76 -3.49 -2  1981 Cross  1066 3007 4399 5701 7007 8507 10307 12712 16486 28  (19) (20) (21) (23) (26) (31) (37) (47) (66) (1										(49)	(74)	(212)
(23) (22) (23) (23) (26) (32) (37) (49) (72) (1 -3.84 -0.78 -1.31 -2.59 -3.41 -2.69 -2.02 -2.75 -3.06 -4 1975 C-,P- 1247 2980 4248 5412 6548 7794 9317 11450 14815 25 (21) (20) (22) (23) (25) (29) (36) (47) (68) (1 -2.83 -9.52 -9.21 -7.11 -6.34 -6.68 -6.67 -4.46 -3.12 -2 1976 P+ 1306 3090 4357 5527 6715 8046 9652 11811 15263 26 (19) (19) (21) (22) (24) (29) (34) (44) (63) (1 2.04 4.00 3.59 3.62 4.76 6.08 6.71 5.58 4.85 3 1977 P+ 1280 3199 4504 5692 6908 8280 9969 12202 15716 26 (22) (19) (21) (22) (25) (30) (36) (46) (64) (1 -0.89 4.04 4.94 5.33 5.51 5.62 6.38 6.12 5.04 2 1978 C+,P+ 1414 3331 4688 5931 7175 8603 10261 12498 16015 26 (23) (24) (26) (27) (32) (35) (41) (54) (75) (1 4.22 4.26 5.53 6.92 6.55 7.06 5.32 4.17 3.03 0 1979 P+ 1357 3327 4739 6018 7331 8775 10509 12875 16534 27 (18) (20) (20) (22) (25) (28) (35) (44) (60) (1 -1.96 -0.14 1.56 2.53 3.87 3.84 4.59 5.38 5.38 3 1980 C-,P- 1220 3155 4542 5823 7108 8572 10365 12706 16239 27 (18) (19) (20) (22) (25) (28) (35) (42) (59) (1 -5.40 -6.29 -6.81 -6.28 -6.38 -5.11 -2.92 -2.76 -3.49 -2					6.55	6.24	6.12	6.18	5.03	4.45	3.46	1.71
1975 C-,P-  1247 2980 4248 5412 6548 7794 9317 11450 14815 25 (21) (20) (22) (23) (25) (29) (36) (47) (68) (1 -2.83 -9.52 -9.21 -7.11 -6.34 -6.68 -6.67 -4.46 -3.12 -2 1976 P+  1306 3090 4357 5527 6715 8046 9652 11811 15263 26 (19) (19) (21) (22) (24) (29) (34) (44) (63) (1 2.04 4.00 3.59 3.62 4.76 6.08 6.71 5.58 4.85 3 1977 P+  1280 3199 4504 5692 6908 8280 9969 12202 15716 26 (22) (19) (21) (22) (25) (30) (36) (46) (64) (1 -0.89 4.04 4.94 5.33 5.51 5.62 6.38 6.12 5.04 2 1978 C+,P+  1414 3331 4688 5931 7175 8603 10261 12498 16015 26 (23) (24) (26) (27) (32) (35) (41) (54) (75) (1 4.22 4.26 5.53 6.92 6.55 7.06 5.32 4.17 3.03 0 1979 P+  1357 3327 4739 6018 7331 8775 10509 12875 16534 27 (18) (20) (20) (22) (25) (28) (35) (44) (60) (1 -1.96 -0.14 1.56 2.53 3.87 3.84 4.59 5.38 5.38 3 1980 C-,P-  1981 Cross  1066 3007 4399 5701 7007 8507 10307 12712 16486 28 (19) (20) (21) (23) (26) (31) (37) (47) (66) (1	1974	C-,P-							9664	11752	15122	25970
1975 C-,P-  1247 2980 4248 5412 6548 7794 9317 11450 14815 25  (21) (20) (22) (23) (25) (29) (36) (47) (68) (1  -2.83 -9.52 -9.21 -7.11 -6.34 -6.68 -6.67 -4.46 -3.12 -2  1976 P+  1306 3090 4357 5527 6715 8046 9652 11811 15263 26  (19) (19) (21) (22) (24) (29) (34) (44) (63) (1  2.04 4.00 3.59 3.62 4.76 6.08 6.71 5.58 4.85 3  1977 P+  1280 3199 4504 5692 6908 8280 9969 12202 15716 26  (22) (19) (21) (22) (25) (30) (36) (46) (64) (1  -0.89 4.04 4.94 5.33 5.51 5.62 6.38 6.12 5.04 2  1978 C+,P+  1414 3331 4688 5931 7175 8603 10261 12498 16015 26  (23) (24) (26) (27) (32) (35) (41) (54) (75) (1  4.22 4.26 5.53 6.92 6.55 7.06 5.32 4.17 3.03 0  1979 P+  1357 3327 4739 6018 7331 8775 10509 12875 16534 27  (18) (20) (20) (22) (25) (28) (35) (44) (60) (1  -1.96 -0.14 1.56 2.53 3.87 3.84 4.59 5.38 5.38 3  1980 C-,P-  1220 3155 4542 5823 7108 8572 10365 12706 16239 27  (18) (19) (20) (22) (25) (28) (35) (42) (59) (1  -5.40 -6.29 -6.81 -6.28 -6.38 -5.11 -2.92 -2.76 -3.49 -2  1981 Cross  1066 3007 4399 5701 7007 8507 10307 12712 16486 28  (19) (20) (21) (23) (26) (31) (37) (47) (66) (1											(72)	(192)
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-2.83 -9.52 -9.21 -7.11 -6.34 -6.68 -6.67 -4.46 -3.12 -2  1976 P+  1306 3090 4357 5527 6715 8046 9652 11811 15263 26  (19) (19) (21) (22) (24) (29) (34) (44) (63) (1  2.04 4.00 3.59 3.62 4.76 6.08 6.71 5.58 4.85 3  1977 P+  1280 3199 4504 5692 6908 8280 9969 12202 15716 26  (22) (19) (21) (22) (25) (30) (36) (46) (64) (1  -0.89 4.04 4.94 5.33 5.51 5.62 6.38 6.12 5.04 2  1978 C+,P+  1414 3331 4688 5931 7175 8603 10261 12498 16015 26  (23) (24) (26) (27) (32) (35) (41) (54) (75) (1  4.22 4.26 5.53 6.92 6.55 7.06 5.32 4.17 3.03 0  1979 P+  1357 3327 4739 6018 7331 8775 10509 12875 16534 27  (18) (20) (20) (22) (25) (28) (35) (44) (60) (1  -1.96 -0.14 1.56 2.53 3.87 3.84 4.59 5.38 5.38 3  1980 C-,P-  1220 3155 4542 5823 7108 8572 10365 12706 16239 27  (18) (19) (20) (20) (22) (25) (28) (35) (42) (59) (1  -5.40 -6.29 -6.81 -6.28 -6.38 -5.11 -2.92 -2.76 -3.49 -2  1981 Cross  1066 3007 4399 5701 7007 8507 10307 12712 16486 28  (19) (20) (21) (23) (26) (31) (37) (47) (66) (1	1975	C-,P-							9317	11450	14815	25249
1976 P+										(47)	(68)	(176)
(19) (19) (21) (22) (24) (29) (34) (44) (63) (1 2.04 4.00 3.59 3.62 4.76 6.08 6.71 5.58 4.85 3 1977 P+ 1280 3199 4504 5692 6908 8280 9969 12202 15716 26 (22) (19) (21) (22) (25) (30) (36) (46) (64) (1 -0.89 4.04 4.94 5.33 5.51 5.62 6.38 6.12 5.04 2 1978 C+,P+ 1414 3331 4688 5931 7175 8603 10261 12498 16015 26 (23) (24) (26) (27) (32) (35) (41) (54) (75) (1 4.22 4.26 5.53 6.92 6.55 7.06 5.32 4.17 3.03 0 1979 P+ 1357 3327 4739 6018 7331 8775 10509 12875 16534 27 (18) (20) (20) (22) (25) (28) (35) (44) (60) (1 -1.96 -0.14 1.56 2.53 3.87 3.84 4.59 5.38 5.38 3 1980 C-,P- 1220 3155 4542 5823 7108 8572 10365 12706 16239 27 (18) (19) (20) (22) (25) (28) (35) (42) (59) (1 -5.40 -6.29 -6.81 -6.28 -6.38 -5.11 -2.92 -2.76 -3.49 -2 1981 Cross 1066 3007 4399 5701 7007 8507 10307 12712 16486 28 (19) (20) (21) (23) (26) (31) (37) (47) (66) (1					-9.21		-6.34	-6.68	-6.67	-4.46	-3.12	-2.77
2.04 4.00 3.59 3.62 4.76 6.08 6.71 5.58 4.85 3  1977 P+	1976	P+						8046	9652	11811	15263	26034
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1980 C-,P-  1220 3155 4542 5823 7108 8572 10365 12706 16239 27  (18) (19) (20) (22) (25) (28) (35) (42) (59) (1  -5.40 -6.29 -6.81 -6.28 -6.38 -5.11 -2.92 -2.76 -3.49 -2  1981 Cross  1066 3007 4399 5701 7007 8507 10307 12712 16486 28  (19) (20) (21) (23) (26) (31) (37) (47) (66) (1											(60)	(156)
(18) (19) (20) (22) (25) (28) (35) (42) (59) (1 -5.40 -6.29 -6.81 -6.28 -6.38 -5.11 -2.92 -2.76 -3.49 -2 1981 Cross 1066 3007 4399 5701 7007 8507 10307 12712 16486 28 (19) (20) (21) (23) (26) (31) (37) (47) (66) (1						2.53	3.87	3.84	4.59	5.38	5.38	3.60
-5.40 -6.29 -6.81 -6.28 -6.38 -5.11 -2.92 -2.76 -3.49 -2  1981 Cross 1066 3007 4399 5701 7007 8507 10307 12712 16486 28  (19) (20) (21) (23) (26) (31) (37) (47) (66) (1	1980	C-,P-							10365	12706	16239	27348
1981 Cross 1066 3007 4399 5701 7007 8507 10307 12712 16486 28 (19) (20) (21) (23) (26) (31) (37) (47) (66) (1												(153)
(19) (20) (21) (23) (26) (31) (37) (47) (66) (1		_							-2.92	-2.76	-3.49	-2.21
	1981	Cross							10307		16486	28453
-5.89 $-5.33$ $-4.87$ $-3.84$ $-2.81$ $-1.54$ $-1.13$ $0.10$ $2.79$ 4												(179)
2117			-5.89	-5.33	-4.87	-3.84	-2.81	-1.54	-1.13	0.10	2.79	4.69

Table I Continued

	Dominance**	Decile Incomes									
Year		1	2	3	4	5	6	7	8	9	10
1982	C- P-	957	2809	4248	5571	6894	8414	10264	12703	16509	28837
	C ,1	(17)	(20)	(22)	(23)	(26)	(31)	(39)	(46)	(67)	(184)
		-4.30	-6.96	-4.92	-3.92	-3.05	-2.10	-0.81	-0.15	0.24	1.49
1983	P+	949	2779	4256	5613	6994	8575	10452	12985	16948	29372
1703	1 '	(17)	(20)	(23)	(24)	(27)	(32)	(38)	(49)	(70)	(180)
		-0.32	-1.03	0.26	1.25	2.65	3.59	3.46	4.21	4.54	2.08
1984	C+.P+	1069	3001	4558	5952	7402	9045	11014	13673	17886	31768
1704	CT,F	(17)	(22)	(24)	(25)	(29)	(34)	(40)	(52)	(74)	(212)
		5.03	7.36	9.23	9.78	10.27	10.10	10.10	9.69	9.20	8.61
1985	D+	1112	3111	4660	6118	7594	9253	11270	14016	18408	32382
1703	1 1	(18)	(22)	(24)	(26)	(30)	(35)	(41)	(54)	(76)	(203)
		1.79	3.51	3.07	4.62	4.65	4.30	4.46	4.58	4.90	2.09
1096	P+	1110	3136	4769	6243	7783	9508	11601	14445	18976	33208
1900	r⊤	(17)	(23)	(25)	(27)	(31)	(35)	(43)	(57)	(79)	(215)
		-0.11	0.77	3.17	3.31	4.43	5.16	5.57	5.46	5.18	2.79
Perce	ent Change										
	′–1986	0.9	21.2	32.9	38.7	42.7	46.1	48.0	49.3	50.5	50.1

<sup>\*</sup>Standard errors in parentheses.

crossings with losses in the bottom decile and significant gains in top deciles. Thus, the bottom decile suffered significant losses in six recession related year-to-year comparisons. In contrast, there does not appear to be a consistent pattern of recessionary impacts on the top decile. In some recession dominated comparisons the mean of the highest decile falls; in others it rises, and in still others it remains unchanged.

Economic Well-Being of Specific Income Classes: Truncated FSD

In a manner analogous to Foster and Shorrocks [10] the information in Table I can be used to evaluate the economic condition of specific income classes across time and to test for differential impacts of growth and change on particular groups. Foster and Shorrocks' method involves applying first degree dominance to the income positions at the lower end of the distribution and using the dominance principle to make general statements about changes in poverty without specifying apoverty cutoff. This amounts to truncating the income distribution and analyzing dominance over a segment. Foster and Shorrocks consider only the bottom of the distribution and use numerical comparisons but their method is easily adapted to other truncations and to the statistical inference procedures used in this paper.

Table II summarizes the test results for improvements and declines in economic well-being, measured at deciles, for three specific groups—the lower, middle, and the upper income classes.

<sup>\*\*</sup>Comparisons to previous year.

<sup>7.</sup> It should be noted that Foster and Shorrocks' [10] FSD analysis of poverty and conditional FSD are related but distinct concepts. They differ in that conditional FSD considers the entire distribution and seeks to make an overall assessment and ranking of well-being. In contrast, the truncated FSD analysis of poverty considers the well-being of the specific group at the bottom of the income distribution and ignores the rest of the population.

**Table II.** Summary of Truncated Stocastic Dominance Comparisons of Year-to-Year Changes in Economic Well-Being by Decile and Income Classes, 1967–1986

	Improved Well-Being	Declining Well-Being	No Change
Poverty			
1st Decile	7	6	6
2nd Decile	9	4	6
3rd Decile	11	4	4
1st and 2nd Decile	9	6	4
Total Poverty Class-1st through 3rd Decile	11	6	2
Middle Class			
4th Decile	12	5	2
5th Decile	13	5	1
6th Decile	13	4	2
7th Decile	13	3	3
Lower Middle Class-4th and 5th Decile	13	5	1
Upper Middle Class-6th and 7th Decile	13	5	3
Total Middle Class-4th through 7th Decile	13	5	1
Upper Class			
8th Decile	13	3	3
9th Decile	15	3	1
10th Decile	11	3	5
Total Upper Class-8th through 10th Decile	15	3	1

We first consider changes in the well-being of the poorest members of society, which involves statistical tests of Foster and Shorrocks' approach to measuring changes in poverty by varying the poverty cutoff. Focusing first on the bottom decile, economic well-being declined in six of 19 comparisons; rose in seven, and remained unchanged in six. Increasing the poverty cutoff to include the second decile and testing for dominance over the two bottom deciles, results in two additional cases where poverty declined. Additional declines are found because two of the year-to-year comparisons in which there was no change in the first decile (1976–77 and 1984–85) had income gains in the second decile. Given equivalence in the bottom decile, dominance in the second decile is sufficient for truncated first degree dominance over the bottom two deciles. Extending the poverty boundary still further to include the third decile adds two additional year-to-year declines in poverty (1985–86).

Application of statistical tests over truncated segments of the income distribution functions reveals that poverty increased in each of the six recession related year-to-year comparisons reported in Table I. This is the case irrespective of whether the poverty boundary is drawn at the first, second or third decile. Further, it is clear from Table I that the income gains of the poorest deciles in the late 1960s and early 1970s evaporated in the post-1979 period.

Truncated dominance can also be applied to evaluate the economic status of the middle class across time. Like the definition of poverty, any delineation of the middle class is somewhat arbitrary. Income cutoffs could be used, but in a manner analogous to Foster and Shorrocks a more reasonable procedure is to truncate the distribution and focus on different cutoffs. We first consider the middle forty percent of income recipients and define the middle class as the fourth through seventh deciles. We then further segment the distribution and separate the fourth and fifth deciles

<sup>8.</sup> This procedure avoids two problems that arise when using specific income cutoffs to define the boundaries of the middle class. First, the quantile groupings, unlike income cut-offs, are not sensitive to the price deflator used. Secondly,

from the sixth and seventh deciles. We refer to these truncated segments of the distribution as the lower middle and upper middle classes respectively. Table II summarizes the rankings obtained by applying the tests for truncated dominance to each middle class decile, the entire middle class and separately to the upper and lower middle classes. For the middle four deciles there were thirteen year-to-year improvements in economic well-being (truncated FSD); five declines, and one annual comparison with no significant change. Thus, what we have defined as the middle class as a whole suffered very significant losses between 1979 and 1982. These losses were recouped between 1983 and 1986.

Recessions have differential impacts on the lower and upper middle class. This is shown in Table II by the difference in the number of year-to-year declines in economic well-being for the fourth and fifth deciles compared to the sixth and seventh deciles. The lower middle class suffered five declines, while the upper middle class experienced only three. Thus, the adverse effects of recessions on the lower middle class are similar to the impacts on the bottom three deciles. In contrast, recessions affect the upper middle class in a manner similar to the impacts on the highest income deciles. Table II shows that the upper income class (deciles eight, nine and ten) experienced 15 year-to-year improvements, suffered three declines and there was one comparison in which none of the upper income deciles had significant changes. Thus, the highest income recipients and the upper middle class are more insulated from the effects of recessions than the lower middle class and those in the bottom three deciles.

#### **IV. Conclusion**

The powerful first degree stochastic dominance theorem provides Paretian partial orders of economic well-being based on income distribution (quantile) functions. We apply the FSD principle and use CPS microdata to test for year-to-year changes in U.S. per capita economic well-being over the period 1967 and 1986. In addition, we consider a more restrictive welfare criterion, conditional FSD, which adds the requirement that an improvement in economic well-being cannot occur unless the incomes of individuals in the poorest decile improve. Finally, we truncate the income distributions and use stochastic dominance to evaluate changes in the status of the lower, middle and upper income classes. Using the original Paretian interpretation of first degree dominance, we are able to rank 17 of 19 year-to-year comparisons. The two years that are not ranked involve recession related comparisons in which incomes fell significantly at the bottom of the distribution and rose significantly at the top.

Using the more restrictive concept of conditional first degree dominance we are able to order 10 of the 19 year-to-year comparisons. Both dominance criteria result in equivalent rankings in the recession related year-to-year declines in economic well-being. However, the Paretian and conditional dominance criteria differ in how they rank well-being during periods of recovery and generally rising incomes. Over the entire two decade period there are nine Paretian improvements but only six of these are ranked as gains in welfare using the conditional dominance criterion. The implications of these differential effects are clear. During downturns in economic activity the individuals occupying the lowest positions in the ordered income distribution in the U.S. always suffer absolute losses, while during periods of prosperity and generally rising incomes they may, but often do not, participate in the improvements in economic well-being. Individuals in upper

whereas income cut-offs can tell us whether the number of family members in the defined middle class is rising or falling, they cannot tell us whether the economic status of the middle class is improving, stable or declining.

income positions are much more insulated from the adverse effects of recessions and participate more fully in the benefits of growth than do those in the bottom positions.

Two other conclusions are noteworthy. We find that the middle class as a whole suffered significant losses in economic well-being between 1979 and 1982 which were recaptured between 1983 and 1986. Recessions are shown to have differential effects on the lower and upper middle income classes, with the former being affected in a manner similar to the poor and the latter having effects similar to the upper income class. Thus, from the perspective of the statistical analysis of U.S. income distributions across time it appears to be more informative to consider two truncated income groups, the bottom and top halves, rather than focusing on the lower, middle and upper income classes.

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