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A SPECTRAL-BASED CUSUM TEST OF EVOLUTIONARY CHANGE

PHILLIP WILD
University of Manchester and CEPM, University of Queensland

We develop a test of evolutionary change that incorporates a null hypothesis of homogeneity, which encompasses time invariance in the variance and autocovariance structure of residuals from estimated econometric relationships. The test framework is based on examining whether shifts in spectral decomposition between two frames of data are significant. Rejection of the null hypothesis will point not only to weak nonstationarity but to shifts in the structure of the second-order moments of the limiting distribution of the random process. This would indicate that the second-order properties of any underlying attractor set has changed in a statistically significant way, pointing to the presence of evolutionary change. A demonstration of the test's applicability to a real-world macroeconomic problem is accomplished by applying the test to the Australian Building Society Deposits (ABSD) model.

Keywords: Cusum Test, Evolutionary Change, Homogeneity, Heterogeneity, Spectral Methods

1. INTRODUCTION

An increasing interest in evolutionary explanations of economic phenomena, including macroeconomic processes and behavior, has emerged over the past decade. For a recent, although brief, survey of this literature, consult Foster and Wild (1999a). Quantitative investigations of such phenomena have been based overwhelmingly on the use of computer-generated simulations of theoretical representations based on nonlinear differential equation systems. Because of their nonlinear form, these equations can produce complex trajectories and attractor sets that contain multiple equilibria, quasi-periodic behavior and even chaos. These equations can also mimic processes of structural change through changes in the topological (qualitative) structure of the underlying attractor set if the investigator deliberately and artificially induces system bifurication [see Haken (1977, 1983), Prigogine (1984), Murray (1989), Coveney and Highfield (1990, ch. 5), Iooss and Joseph (1990), Zhang (1991), and Foster and Wild (1996)].

There has been, in contrast, relatively little work undertaken on establishing an econometric framework or methodology that is capable of explicitly detecting and modeling evolutionary economic explanations of macroeconomic processes.

Address correspondence to: Dr. Phillip Wild, CEPM, School of Economics, University of Queensland, Brisbane, QLD, 4072, Australia; e-mail: P.wild@economics.uq.edu.au.

One factor that has complicated such attempts involves the definition of evolution. The term evolution means different things in different academic disciplines; for example, the term evolution is associated with nonlinearity in functional form in both the natural sciences and broader evolutionary economics literature [consult Haken (1977, 1983, 1988), Weidlich and Haag (1982), Murray (1989), Coveney and Highfield (1990), Weidlich (1991), Zhang (1991), and Foster and Wild (1996)). On the other hand, in the time-series literature, the term evolution has been equated with nonstationarity following the seminal work of Priestley (1965).

Conventional evolutionary economic interpretations emphasize model frameworks and explanations that are embedded in a self-organizational approach to system dynamics. Such explanations stress the importance of dissipative structures that exhibit structural development through parallel increases in organization and complexity toward capacity limits [Foster and Wild (1999a, p. 111)]. These processes possess the sigmoid shape conventionally associated with logistic processes and can subsequently be modeled as logistic processes, possibly with augmentation to include exogenous effects and endogenous interactive effects; see, for example, Foster and Wild (1996, 1999a, 1999b). In this particular case, the validity of the usual equilibrium/disequilibrium dichotomy that views time-series data as a series of disequilibrium points between static equilibrium states is questionable because of the inherent structural instability that arises as the capacity limit is approached. Instead, such processes have emergent critical tendencies associated with a "narrowing basin of attraction" during the saturation stage of the process, leading to structural discontinuities associated with basin "spillovers" that can generate new equilibrium states with different qualitative properties. Preliminary attempts at devising an explicit econometric framework that can detect and model such processes are presented by Foster (1992, 1994), Burley and Foster (1994), and Foster and Wild (1999a, 1999b).

In more general terms, however, it is possible to interpret the term evolution as conveying one or more generic and distinguishing properties such as time irreversibility. It then follows that a close correspondence will exist between the objective of detecting and modeling evolutionary processes and many developments that have occurred in the econometric/time-series literature. For example, if we adopt the time-series convention of equating evolutionary change with nonstationarity, then a close correspondence will exist between attempts to detect and model evolutionary processes and the literature on nonstationarity, time-varying models and estimation procedures, structural change, and regime-shifting models. In the next section, the term "evolutionary change" is formally equated with weak nonstationarity, which involves time variation in the variance and/or autocovariance structure that could be associated with the emergence of different phases or regimes of a process.

A general econometric methodology would have to encompass two different but related aspects. The first is the development of a broad model framework that would enable investigators to detect evolutionary change. This is a central task because it would enable an assessment of whether the "non-evolutionary" techniques that permeate both macroeconomic and econometric theory are valid approaches. The second task is fundamentally more difficult and involves the actual modeling of evolutionary processes. This latter task is difficult because of the extreme difficulties that arise in attempting to address complex, nonlinear theory with econometric methods as acknowledged, for example, by Granger (1993). Further complicating evolutionary modeling is the fact that there is no generic definition of nonstationarity in the time-series literature, complicating attempts to model processes with time-dependent structures. Most effort has probably been expended upon the first task without most investigators necessarily associating their work with the investigation of evolutionary phenomena, as broadly defined. In this paper, we focus on the first aspect as well, that is, on the *detection of evolutionary change*.

The structure of this paper is as follows: In the next section, certain methodological considerations are addressed. This is followed in Section 3 with the development of an individual frequency test for homogeneity, culminating in a joint Cusum test of homogeneity, termed the *aggregate homogeneity (AH) test*. In Section 4, applicability of the test procedure is demonstrated by applying it to the Australian Building Society Deposits (ABSD) example considered by Foster and Wild (1999a). Section 5 of the paper contains some concluding comments.

2. SOME METHODOLOGICAL CONSIDERATIONS

In Foster and Wild (1999a), it is argued that evolutionary explanations of historical economic processes can be expected to have two components—a deterministic component and a nondeterministic or stochastic component. The econometric framework adopted by investigators should include both components. In this paper, we restrict attention to univariate models.[1] Heuristically, the following econometric framework is proposed:

$$Y_t = f\left(\boldsymbol{X}_t^{\boldsymbol{T}}, \boldsymbol{\beta}\right) + \nu_t, \tag{1a}$$

$$\nu_t = \sum_{u=0}^{\infty} g_u \varepsilon_{t-u}, \tag{1b}$$

where Y_t is a dependent variable, $\boldsymbol{\beta}$ is a vector of parameters, $\boldsymbol{X_t}$ is a vector of regressor variables, f is a function operator, ν_t is a disturbance, $\varepsilon_t \approx NI(0, \sigma^2)$, and $\{g_u\}$ is a given sequence of constants satisfying $\sum_{u=0}^{\infty} g_u^2 < \infty$. If we assume that the vector of regressor variables is deterministic, then the first part of (1a), $f(\boldsymbol{X}_t^{\boldsymbol{T}}, \boldsymbol{\beta})$, can be interpreted as capturing the "deterministic" part of the process while the second part of (1a), ν_t, which is explicitly modeled by (1b), can be interpreted as capturing the "stochastic" component of the economic process. Even if (1a) contains lagged values of the dependent variable Y_t, these variables can be regarded as fixed and given from the perspective of what variables are directly observable in the current time period t even though they are determined by a stochastic process. Furthermore, any dependence introduced by the inclusion of such variables will be explicitly captured in (1b).

If $f(\boldsymbol{X}_t^T, \boldsymbol{\beta})$ in (1a) contains stochastic regressors, then the deterministic/stochastic dichotomy mentioned above will break down. In this case, the first part of (1a) can be viewed as capturing the average behavior of the model with the averages being given by probability limits. Equation (1b) can be viewed as capturing deviations from this average behavior. The methodological discussion that follows is not sensitive to whatever dichotomy is adopted—the main implication associated with adopting either the deterministic/stochastic or average/deviation interpretation relates to the different terminology one must adopt.

The structure of (1a) as determined by f is deliberately left general but could include a nonlinear functional form.[2] The only restriction, per se, is that the deterministic part of the model is stationary in parameters.[3]

Equation (1b) models that part of the process not explained by the deterministic model. For the purpose of detecting evolutionary change, it is sufficient that the random process modeled in (1b) be restricted to a stationary general linear process [see Priestley (1981, pp. 141–147)]. This representation then admits the most general form of linear stationary time-series model identified in the time-series literature. We define a non-evolutionary process as a process that admits the representation outlined in (1a) and (1b), above, with the added proviso that the process has a time-reversible probabilistic structure.[4] In general, model frameworks that utilize fixed coefficient linear stationary structures such as (1) can be viewed fundamentally as "non-evolutionary." Estimation schemes based on such models, such as OLS, Yule-Walker, and MLE, can be viewed similarly as "non-evolutionary filters."[5]

Given the econometric framework outlined in (1), a very broad definition of evolutionary change is proposed. This definition encompasses the definition of evolution adopted in the broader evolutionary economics literature which emphasizes the role of nonlinearity in functional form [see Zhang (1991) and Foster and Wild (1996, 1999a)], as well as the definition of evolution adopted in the time-series literature, which equates evolution with nonstationarity [see Priestley (1965, 1981, 1988) and Foster and Wild (1995)].

Formally, statistical definitions of evolutionary change will center around the existence of time-irreversible structure associated with one of three potential generating mechanisms. The first mechanism arises when time-irreversible structure is generated by nonlinearity in the functional form of (1a), given that the other assumptions made about (1a) and (1b) hold—that is, (1a) is stationary in parameters and (1b) is a stationary general linear process. The second mechanism generating time irreversibility arises when the functional form of (1a) is linear in variables and linear and stationary in parameters but (1b), while being stationary in parameters, has a non-Gaussian distribution. These two particular scenarios have been dealt with by Hinich and Rothman (1998). In this context, note that because nonlinearity and non-Gaussianity do not imply time irreversibility, one has to directly establish whether the process is time irreversible by employing tests like those documented by Ramsey and Rothman (1996) and Hinich and Rothman (1998), in addition to conventional tests of Gaussianity and linearity as outlined by Hinich (1982), Subba Rao and Gabr (1984), and Ashley et al. (1986), for example. If the process is time irreversible, then this is conclusive proof that it is evolutionary.

The third source of time irreversibility arises when (1a) or (1b) is nonstationary in parameters. In this context, note that parameter variation in (1b) might ultimately reflect parameter variation in (1a) that is being captured in the residuals of estimated relationships. In what follows, we assume, however, that (1a) is stationary in parameters. Therefore, we restrict the possibility of parameter variation to (1b).

It is argued by Foster and Wild (1999a) that if the data-generating mechanism (DGM) is evolutionary, then the presence of nonlinearity and/or nonstationarity in the unexplained part of the process is *unavoidable* because the full intricacies of these interactions cannot be completely modeled by the deterministic part of the process represented by (1a). We focus on the third broad generating mechanism—that is, where (1b) is nonstationary in parameters. We develop a test that is capable of establishing whether (1b) is stationary in parameters. This constitutes a clear restriction on both the definition and the type of evolutionary change that can be detected by the test statistic developed in this paper; we are essentially adopting the conventional time-series definition of evolution that equates evolution with nonstationarity.

In developing this test, we adopt what is essentially an indirect approach. Specifically, we make use of the fact that there is a correspondence between the sequence of constant coefficients $\{g_u\}$ and autocovariance generating function (and therefore position and shape of the power spectrum) for the class of stationary general linear processes. In fact, this correspondence is unique if the process is not only stationary but also invertible [see Box and Jenkins (1970, p. 195) and Priestley (1981, pp. 145–146)]. This uniqueness property, however, is not central to our argument. Even if the process is not invertible, statistically significant changes in spectral position or shape will still point to parameter variation and evolutionary change in (1b).

Given that our stated aim is to detect evolutionary change, model (1) is non-evolutionary if the error structure outlined in (1b) is homogeneous. This implies that the variance/autocovariance structure of (1b) is stationary (time invariant). In the time-series literature, both time- and frequency-domain tests of homogeneity have been developed. The time-domain tests have centered around testing whether the residual variance from regression models is constant [Brown et al. (1975)]. In the frequency domain, however, tests of homogeneity have been associated with tests of uniformity in spectral shape and have tended to focus on shifts in autocovariance structure. These tests have been based around comparison of spectral density estimates, Kolmogorov-Smirnov tests applied to normalized integrated (cumulative) spectra, and Kullback-Leibler spectral divergence tests that are applied to spectral ratios; for example, see Jenkins (1961), Priestley and Subba Rao (1969), Priestley (1981, pp. 475–488; 1988), Durlauf (1991), and Fong and Ouliaris (1995).

The concept of heterogeneity (or evolutionary change) developed in this paper encompasses both of the preceding time-series definitions. Specifically, shifts in variance and autocovariance structure, through their effect on both the *magnitude* and the *shape* of power spectra, will constitute legitimate forms of heterogeneity, which can be detected, in principle, by the test statistic developed in this paper.

The type of heterogeneity being addressed in this paper, and implications for statistical inference, closely follow concerns expressed by Keynes in his famous debate with Tinbergen in the late 1930's and early 1940's about the role of Econometrics; see Keynes (1939) and Tinbergen (1940). Keynes argued that:

> Put broadly, the most important condition is that the environment in all relevant respects, other than the fluctuations in those factors of which we take particular account, should be uniform and homogeneous over a period of time. We cannot be sure that such conditions will persist in the future, even if we find them in the past. But if we find them in the past, we have at any rate some basis for an inductive argument. [Keynes (1939, p. 566)]

Furthermore, he not only stressed the importance of homogeneity, but proposed the following testing scheme:

> The first step, therefore, is to break up the period under examination into a series of sub-periods, with a view to discovering whether the results of applying our method to the various sub-periods taken separately are reasonably uniform. If they are, then we have some ground for projecting our results into the future. [Keynes (1939, pp. 566–567)]

This procedure was viewed by Keynes as a way of testing whether the underlying properties of the estimated process were statistically stable [Keynes (1973, ch. 32)].

In the time-series literature, this concept of uniformity is very closely related to a particular concept of stationarity, which has been termed *relative stationarity* by Parthasarathy and Varadhan (1964, pp. 68–69) and Brillinger (1981, p. 22). The homogeneity test statistic developed in this paper can be viewed, alternatively, as a test of *weak relative stationarity*. From a theoretical perspective, the importance of the concept of relative stationarity follows from the fact that, given the additional assumption of stochastic continuity, we know that a stationary extension of the process exists over all time [Parthasarathy and Varadhan (1964, p. 68), Brillinger (1981, p. 22)]. In this respect, the definition of a stationary extension implies that there exists a process defined for all time whose distribution would coincide with the distribution of our observed process when the time index of the former process is limited to the one corresponding to the observed sample period [Parthasarathy and Varadhan (1964, p. 65)]. The clear implication of Parthasarathy and Varadhan's work is that if the process is *not* relatively stationary over the sample period, then a stationary extension will not exist, ultimately bringing into question the validity of asymptotic theory based upon the concept of asymptotic stationarity which fundamentally underpins the notion of limiting distribution.

Associated with this observation is the further fact that definitions of asymptotic stationarity are predicated upon the assumption of a fixed parametric structure; see

Priestley (1981, pp. 121–147), for example. This implies, for the class of stationary general linear processes, a constant (stationary) autocovariance structure that would require, in turn, the fulfilment of the relative stationarity condition. Otherwise, the correspondence between the parametric structure and the autocovariance structure would be violated, leading to parameter variation and evolutionary change in (1b).

The violation of the concept of relative stationarity and associated change in the limiting distribution of the random process modeled by (1b) or, equivalently, with the associated parameter variation in (1b), are consistent, in principle, with the notion of moving equilibrium or changing basin of attraction mentioned by Foster and Wild (1999a). In this respect, changes in the limiting distribution would reflect statistically changes in the equilibrium properties of the process, pointing to the presence of evolutionary change.

It is also evident from conventional time-series definitions of time reversibility that evolutionary change in (1b) implies time irreversibility because evolutionary change in (1b) implies weak nonstationarity. This fact follows from conventional definitions of time reversibility because it is possible to show that time reversibility implies stationarity and nonstationarity implies time irreversibility [see Lawrance (1991), Ramsey and Rothman (1996), and Hinich and Rothman (1998)]. Therefore, evidence of evolutionary change in (1b) involving the violation of the relative stationarity condition will, in principle, confer on the random process being modeled by (1b) properties conventionally associated with evolutionary economic processes, namely, a changing equilibrium structure and a time-irreversible probabilistic structure.

The type of testing procedure explicitly advocated above by Keynes is quite different from the standard testing procedures based on auxiliary regressions or sequential techniques such as recursive least squares (RLS). The type of heterogeneity alluded to above by Keynes is more likely to be directly detected from the use of moving window regression techniques [see Fama and MacBeth (1973), Brown et al. (1975), Ploberger and Kramer (1992), Andrews (1993), Chu et al. (1995), and Foster and Nelson (1996)]. Investigators, however, have employed the rolling window method as primarily a graphical or descriptive device, with particular emphasis on assessing variation in estimated coefficients.[6]

There is also a question mark over the ability of conventional diagnostic tests to detect certain forms of heterogeneity that involve smooth (or subtle), but ultimately significant, time variation in structure associated with different "phases" or regimes. For example, Foster and Wild (1999a) found evidence pointing to the existence of heterogeneity involving changing patterns of serial dependence in a set of residuals that passed all of the standard diagnostic tests activated in the PC-Give 8 econometric package [Doornik and Hendry (1994)]. This evidence was based on applying the concept of evolutionary spectra [consult Priestley (1965, 1981, 1988), Cohen (1989), Artis et al. (1992), and Foster and Wild (1995)]. These authors found that the power spectra associated with the first frame of 100 observations of the standardized residuals from the estimated equation appeared to be qualitatively

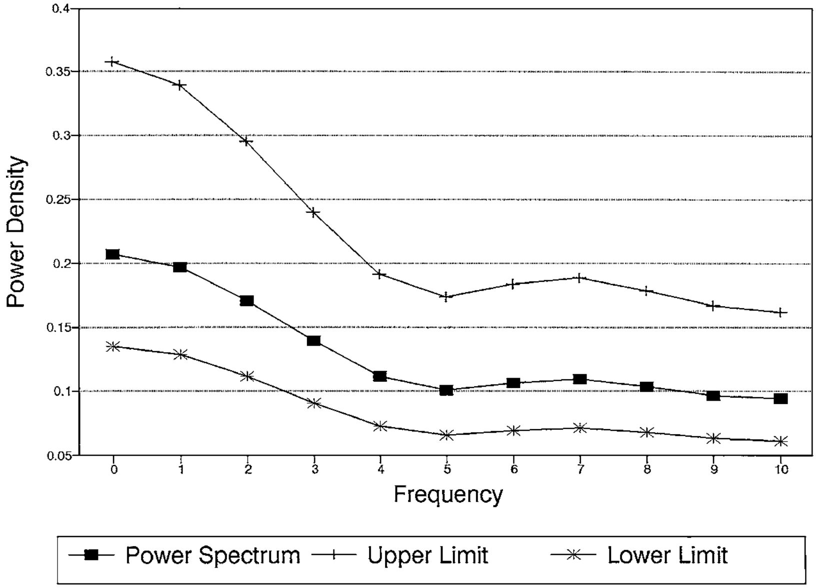

Figure 1. Plot of power spectrum in frame 1, frame size = 100.

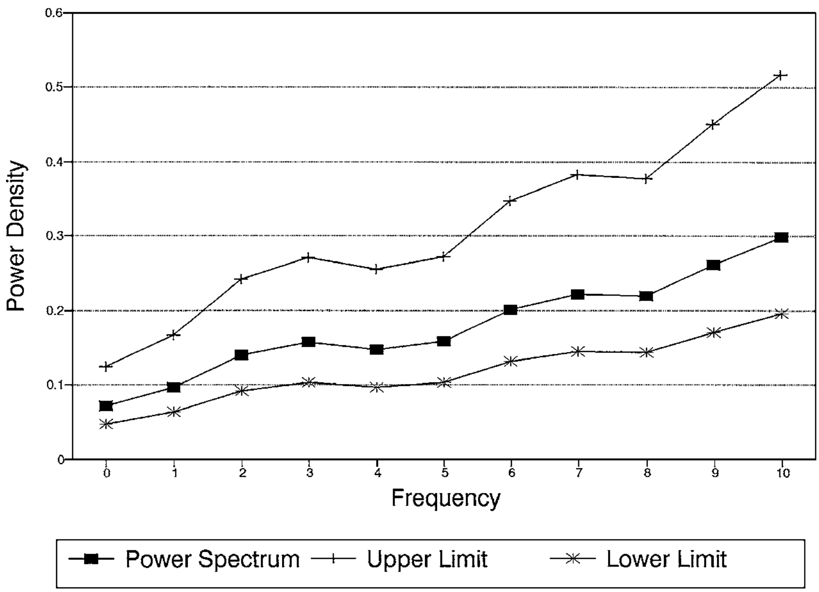

Figure 2. Plot of power spectrum in frame 151, frame size = 100.

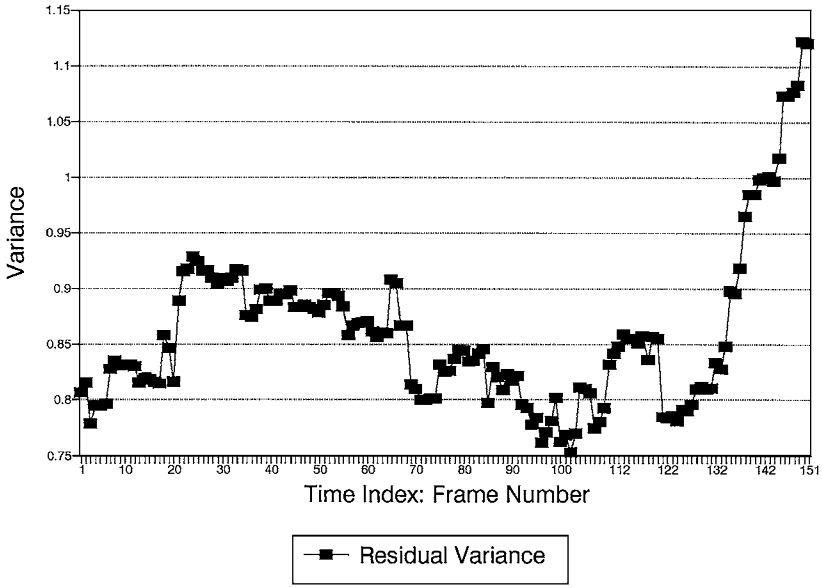

FIGURE 3. Time path of residual variance, frame size = 100.

different from the power spectra of a *non-overlapping* frame containing the last 100 observations (frame 151). Plots of these two power spectra with 95% confidence intervals are contained in Figures 1 and 2. These same authors also found evidence of significant trends in the time path of the residual variance; see Foster and Wild (1999a, p. 127) and Figure 3.

The test that is developed in the next section of the paper will formalize the ideas of Foster and Wild (1996, 1999a, 1999b) by attempting to rigorously verify statistically the qualitative change alluded to in the aforementioned papers. The test also builds upon the aggregate test outlined by Wild (1996).

3. AGGREGATE HOMOGENEITY (AH) TEST STATISTIC

The objective of the test is to determine whether the spectral decomposition associated with one frame is the same as the spectral decomposition associated with another frame in a statistical sense.[7] This means that any variation in spectral decomposition between these two frames should be able to be attributed to random chance. If the variation is greater than what can be reasonably attributed to random chance, then evidence of heterogeneity and nonstationarity in the second-order moment structure of the process will be forthcoming.

In Wild (1996), the following test statistic is proposed:

$$\boldsymbol{ts(k) = f(i, k) - f(j, k)} \tag{2}$$

where $\boldsymbol{f}(\boldsymbol{i}, \boldsymbol{k})$ is the nonnormalized spectral density estimate for frame i and frequency k, and with variables i and j denoting particular data frames. The formulas used to estimate $\boldsymbol{f}(\boldsymbol{i}, \boldsymbol{k})$ are listed in the Appendix.

In calculating the test statistic, we use natural logarithms of spectral density estimates because this transformation ensures that each transformed spectral estimate will have equal variance [Rao and Shapiro (1970, p. 211)]. Therefore, denote each transformed spectral density estimate by capital letters; that is,

$$\boldsymbol{F}(\boldsymbol{i}, \boldsymbol{k}) = \ell\mathrm{n}[\boldsymbol{f}(\boldsymbol{i}, \boldsymbol{k})], \tag{3}$$

with (2) now becoming

$$\boldsymbol{TS}(\boldsymbol{k}) = \boldsymbol{F}(\boldsymbol{i}, \boldsymbol{k}) - \boldsymbol{F}(\boldsymbol{j}, \boldsymbol{k}). \tag{4}$$

The methodology underpinning the test procedure is as follows. If the difference $\boldsymbol{TS}(\boldsymbol{k})$ is small relative to its standard deviation, then no significant change between the two frames would have occurred at frequency k. However, if the difference is large relative to its standard deviation, then it is likely that structural change has occurred between the two frames at frequency k, indicating the presence of heterogeneity and evolutionary change. Therefore, to apply the test, we need to determine the probability distribution of the change in spectral decomposition between the two frames at frequency k.

The distribution theory of the individual frequency test is based on standard distribution theory associated with spectral density estimates. The following assumptions will guarantee the properties we require in order to develop the individual frequency test statistic:

Assumption 1. The process is assumed to have an absolutely summable autocovariance structure.

Assumption 2. All moments up to at least the fourth order exist, and are stationary.

Assumption 1 is a finite-dependence (or mixing) assumption and guarantees the existence and continuity of the nonnormalized spectral density function for all frequencies w in the principal domain $(-\pi, \pi)$ [Parzen (1957, p. 330), Brillinger (1981, p. 24), and Priestley (1981, p. 416)]. Note in particular that Assumption 1 rules out the possibility of singularities associated with long memory processes as dealt with by Hidalgo (1996), for example. Assumption 2 contains the necessary higher-order moment restrictions required to prove the asymptotic normality of spectral estimates while also ensuring that the fourth-order cumulant structure is absolutely summable—a condition required in the derivation of asymptotic expressions for variance and covariance of spectral estimates [see Parzen (1957, pp. 332–333), Anderson (1971, pp. 531, 534), Priestley (1981, p. 469) and Andrews (1991, pp. 823–824)].

The periodogram can be viewed as a sample version of the nonnormalized spectral density function. It can be shown that for each (and all) frequencies w in the principal domain $(-\pi, \pi)$, the periodogram is an asymptotically unbiased but inconsistent estimator of the nonnormalized spectral density function [Priestley (1981, pp. 417, 420–421)].

To construct consistent estimates of the nonnormalized spectral density function $\boldsymbol{h}(\boldsymbol{w})$, lag windows have been conventionally used. Under suitable regularity conditions on the window functions, we can obtain a consistent estimate of the nonnormalized spectral density function [Brillinger and Rosenblatt (1967, pp. 157–158), Brillinger (1981, pp. 95, 123–125), and Priestley (1981, p. 451)]. The Parzen lag window employed in empirical work in this paper satisfies these conditions [Priestley (1981, p. 451)].

We then verify that, for scale parameter windows such as the Parzen window,

$$\lim_{N\to\infty} E\{\hat{\boldsymbol{h}}(\boldsymbol{w})\} = \boldsymbol{h}(\boldsymbol{w}), \qquad \text{all } w \tag{5}$$

$$\lim_{N\to\infty} \left[\frac{N}{M}\operatorname{Var}\{\hat{\boldsymbol{h}}(\boldsymbol{w})\}\right] = (1+\boldsymbol{\delta k})\boldsymbol{h}^2(\boldsymbol{w})\frac{1}{N}\left[\sum_{s=-(N-1)}^{(N-1)} \lambda^2(s)\right], \tag{6}$$

and

$$\lim_{N\to\infty} \left[\frac{N}{M}\operatorname{cov}\{\hat{\boldsymbol{h}}(\boldsymbol{w}_1), \hat{\boldsymbol{h}}(\boldsymbol{w}_2)\} = 0\right], \qquad w_1 \neq \pm w_2, \tag{7}$$

where N is the sample size, M is a window truncation parameter, and $\boldsymbol{h}^2(\boldsymbol{w})$ is the second-order derivative of the true spectral density function at frequency w [consult Brillinger and Rosenblatt (1967, p. 159), Anderson (1971, p. 531), Brillinger (1981, pp. 135–136, 143–145, 147–151), and Priestley (1981, pp. 454–457)].

The asymptotic normality of spectral estimates can be established under the conditions listed in Assumptions 1 and 2 [consult Anderson (1971, pp. 534–540), Brillinger (1981, p. 149), and Priestley (1981, p. 469)]. In general, given Assumptions 1 and 2, spectral estimates at any fixed number of frequencies will have a limiting joint normal distribution with variances and covariance given by (6) and (7). It also follows as a consequence of both asymptotic Gaussianity and (7) that the spectral estimates also will be asymptotically independent. Furthermore, because the "raw" spectra are asymptotic independent normal random variates, the logarithmically transformed spectra will also be asymptotic independent normal random variates [Priestley (1981, p. 471)].

Note that the statistical theory developed in this paper depends crucially on the assumption that spectral density estimates have an asymptotic normal distribution. From the perspective of large sample theory, this is nonstandard. In the standard literature, the asymptotic distribution of the raw periodogram is chi square with two degrees of freedom for $w \neq 0$ or π, and chi square with one degree of freedom if $w = 0, \pi$. If the raw periodogram is smoothed using one of the standard lag windows, then the asymptotic distribution will be chi square with the appropriate degrees of freedom given by each window's respective equivalent degrees of freedom ν, see Table 6.2 in Priestley (1981, pp. 466–467). In the case of the Parzen lag window, $\nu = 3.708614 * (N/M)$. For standard windows such as the Parzen window, $(N/M) \to \infty$, so that $\nu \to \infty$ as $N \to \infty$. The normality assumption is valid asymptotically because the χ^2_ν distribution tends to a normal distribution as $\nu \to \infty$; see Priestley (1981, p. 469) and Diggle (1990, pp. 103–104).

In large sample approximations, the normal distribution will only be a good approximation if the effective degrees of freedom ν are relatively large in magnitude. This is more likely to be the case if the sample size N is much larger than the window truncation parameter M. This, in turn, will produce an estimate of the spectral density that is very smooth, which requires, in turn, that the true spectral density be smooth. Furthermore, if N is large relative to M, then neighboring frequencies are likely to be more correlated rather than independent. It will certainly be the case that the smaller is the value of M, the larger will be the bandwidth of the window, requiring that two frequencies must be separated by a greater magnitude if they are to be approximately uncorrelated; see Priestley (1981, pp. 456–457, 527). This could mean that some frequencies might have to be skipped in forming the Cusum test statistic.[8]

The $\boldsymbol{TS}(\boldsymbol{k})$ statistic will be asymptotically independently normally distributed because the statistic is, by construction, a linear combination of the $\boldsymbol{F}(\boldsymbol{i}, \boldsymbol{k})$'s which are themselves asymptotically independently normally distributed variates. We now verify that the mean and variance of the $\boldsymbol{TS}(\boldsymbol{k})$ statistic are 0 and $(Km/N)(1 + \boldsymbol{\delta}_k) * \{2 - 2 * \boldsymbol{R}_j\}$, respectively.

We prove that $E[\boldsymbol{TS}(\boldsymbol{k})] = 0$ holds identically.[9] To prove this, we use the result that

$$E[\boldsymbol{F}(\boldsymbol{i}, \boldsymbol{k})] = F_k + \frac{6}{M^2}\boldsymbol{h}^2(\boldsymbol{w}) + O\left(\frac{\log N}{N}\right), \tag{8}$$

for frame i, where the term $\boldsymbol{F}(\boldsymbol{i}, \boldsymbol{k})$ is the spectral density estimate associated with frame i and frequency k, and F_k is the *true* spectral density estimate at frequency k. The term $(6/M^2)\boldsymbol{h}^2(\boldsymbol{w})$ denotes the asymptotic expression for bias associated with the Parzen window with the term $\boldsymbol{h}^2(\boldsymbol{w})$ denoting the second-order derivative of the true spectral density function at frequency w. The term $O(\log N/N)$ denotes the bias attributable to the periodogram itself [see Priestley (1981, pp. 458, 462) and Hinich (1994)].[10] We assume that the same smoothing kernel is applied to each frame. Given equation (8), equation (4) gives

$$\begin{aligned} E[\boldsymbol{TS}(\boldsymbol{k})] &= E[\boldsymbol{F}(\boldsymbol{i}, \boldsymbol{k})] - E[\boldsymbol{F}(\boldsymbol{j}, \boldsymbol{k})] \\ &= F_k + \frac{6}{M^2}\boldsymbol{h}^2(\boldsymbol{w}) + O\left(\frac{\log N}{N}\right) - \left[F_k + \frac{6}{M^2}\boldsymbol{h}^2(\boldsymbol{w}) + O\left(\frac{\log N}{N}\right)\right] \\ &= 0. \end{aligned} \tag{9}$$

The "blurring" effect due to the bias in finite samples associated with the use of the Parzen lag window and the periodogram itself cancel, as do the F_k's under the null hypothesis of homogeneity, giving a difference of zero. Therefore, (9) states that the bias will cancel provided that two factors are fulfilled: First, the shape and, therefore, curvature of the true spectra are the same, which is assumed under the null hypothesis of homogeneity. Second, the same smoothing kernel (and therefore parameters M and N) are applied to each frame.

The variance of the $\boldsymbol{TS}(\boldsymbol{k})$ statistic, denoted $\text{Var}[\boldsymbol{TS}(\boldsymbol{k})]$ is given by

$$\text{Var}[\boldsymbol{TS}(\boldsymbol{k})] = \text{Var}[\boldsymbol{F}(\boldsymbol{i},\boldsymbol{k}) - \boldsymbol{F}(\boldsymbol{j},\boldsymbol{k})]. \qquad \textbf{(10)}$$

This follows from the fact that $\boldsymbol{TS}(\boldsymbol{k})$ is, for each k, a linear combination of random variates because both $\boldsymbol{F}(\boldsymbol{i},\boldsymbol{k})$ and $\boldsymbol{F}(\boldsymbol{j},\boldsymbol{k})$ are random variates. We then use the formula

$$\text{Var}(X \pm Y) = \text{Var}(X) + \text{Var}(Y) \pm 2\,\text{Cov}(X, Y), \qquad \textbf{(11)}$$

and express (10) as

$$\text{Var}[\boldsymbol{TS}(\boldsymbol{k})] = \text{Var}[\boldsymbol{F}(\boldsymbol{i},\boldsymbol{k})] + \text{Var}[\boldsymbol{F}(\boldsymbol{j},\boldsymbol{k})] - 2 * \text{Cov}\{\boldsymbol{F}(\boldsymbol{i},\boldsymbol{k}), \boldsymbol{F}(\boldsymbol{j},\boldsymbol{k})\}. \qquad \textbf{(12)}$$

We use the following expressions for asymptotic variance and covariance [Rao and Shapiro (1970, p. 216)]:

$$\text{Var}[\boldsymbol{f}(\boldsymbol{i},\boldsymbol{k})] =^{a} (KM/N) f_k^2 (1 + \boldsymbol{\delta_k}), \qquad \textbf{(13a)}$$

$$\text{Cov}[\boldsymbol{f}(\boldsymbol{i},\boldsymbol{k}), \boldsymbol{f}(\boldsymbol{j},\boldsymbol{k})] =^{a} (KM/N)[\boldsymbol{c}^2(i, j)k + \boldsymbol{q}^2(i, j)k](1 + \boldsymbol{\delta_k}), \qquad \textbf{(13b)}$$

where the symbol $\overset{a}{=}$ denotes "asymptotically equal to," K is dependent upon the particular lag window adopted in spectral estimation, and $c^2(i, j)k$ and $q^2(i, j)k$ are cospectral and quadrature spectral estimates at frequency k for the pair of series $X_i, \ldots, X_{i+N-1}$ and $X_j, \ldots, X_{j+N-1}$ respectively.[11] The term δ_k takes the values

$$\boldsymbol{\delta_k} = \begin{cases} 1 & \text{if } k = 1 \text{ or } MF + 1 \\ 0 & \text{otherwise} \end{cases}$$

[see Priestley (1981, p. 455)], where parameter $MF + 1$ is an integer that denotes the total number of individual frequencies that are to be estimated.

Using the arguments of Rao and Shapiro (1970, p. 216), we obtain, after taking the natural logarithm of spectral estimates,

$$\textbf{Var}\,[\boldsymbol{F}(\boldsymbol{i},\boldsymbol{k})] \overset{a}{=} (KM/N)(1 + \boldsymbol{\delta}_k), \qquad \textbf{(14a)}$$

$$\begin{aligned} \text{Cov}[\boldsymbol{F}(\boldsymbol{i},\boldsymbol{k}), \boldsymbol{F}(\boldsymbol{j},\boldsymbol{k})] &\overset{a}{=} \frac{KM}{N} \frac{c^2(i, j)k + q^2(i, j)k}{f_k^2}(1 + \boldsymbol{\delta}_k) \\ &\overset{a}{=} (KM/N)(1 + \boldsymbol{\delta}_k) * \boldsymbol{R}^2(\boldsymbol{i}, \boldsymbol{j})\boldsymbol{k}, \qquad \textbf{(14b)} \end{aligned}$$

where $\boldsymbol{R}^2(\boldsymbol{i}, \boldsymbol{j})\boldsymbol{k} = \boldsymbol{R}_j$ is the coherence between the series (i.e., between frames i and j) at frequency k.[12] The formulas used to estimate $c(i, j)k$, $q(i, j)k$, and $\boldsymbol{R}^2(\boldsymbol{i}, \boldsymbol{j})\boldsymbol{k}$ are listed in the Appendix.

Now, substituting (14a) and (14b) into (12), we obtain

$$\mathrm{Var}[\boldsymbol{TS}(\boldsymbol{k})] \stackrel{a}{=} (KM/N)(1+\boldsymbol{\delta}_k) + (KM/N)(1+\boldsymbol{\delta}_k) - 2*(KM/N)(1+\boldsymbol{\delta}_k)*\boldsymbol{R}_j. \quad \textbf{(15)}$$

Collecting the common term $(KM/N)(1+\boldsymbol{\delta}_k)$, we obtain

$$\mathrm{Var}[\boldsymbol{TS}(\boldsymbol{k})] \stackrel{a}{=} (KM/N)(1+\boldsymbol{\delta}_k)*\{2-2*\boldsymbol{R}_j\}. \quad \textbf{(16)}$$

To activate the test, it is assumed that the desired resolution bandwidth is known by the investigator. In practical terms, this will mean that the following "parameter" settings have been predetermined: the frame length (N), the number of frequencies to be estimated ($MF+1$), and the truncation parameter used in the lag window (M).

The following steps are then performed:

1. Calculate the variance for each frequency k from (16), which is termed $\boldsymbol{VARF}(\boldsymbol{k})$.
2. Compute the natural logarithms of the spectral estimates for frames i and j.
3. Compute the difference $\boldsymbol{TS}(\boldsymbol{k})$:

$$\boldsymbol{TS}(\boldsymbol{k}) = \boldsymbol{F}(\boldsymbol{i},\boldsymbol{k}) - \boldsymbol{F}(\boldsymbol{j},\boldsymbol{k}). \quad \textbf{(17)}$$

   and square it.
4. Compute the test statistic $\boldsymbol{TSS}(\boldsymbol{k})$:

$$\boldsymbol{TSS}(\boldsymbol{k}) = \boldsymbol{TS}(\boldsymbol{k})^2 \,/\, \boldsymbol{VARF}(\boldsymbol{k}), \quad \textbf{(18)}$$

where $\boldsymbol{TS}(\boldsymbol{k})$ is computed in step (3) and $\boldsymbol{VARF}(\boldsymbol{k})$ in step (1).

Under the null hypothesis of homogeneity, the $\boldsymbol{TSS}(\boldsymbol{k})$ statistic will be, for each frequency k, an asymptotically independently normally distributed variate. Moreover, the form of the test statistic outlined in step (4) is a squared standard normal variate. As such, it will have, for each frequency k, a chi-square distribution with one degree of freedom [see Rao and Shapiro (1970, p. 218)].

The joint test statistic we consider is a Cusum test statistic, termed the *aggregate homogeneity* (AH) test statistic, and is obtained by *summing* the individual test statistics $\boldsymbol{TSS}(\boldsymbol{k})$ [cf. equation (18)], for all k; that is,

$$AH = \sum_{k=1}^{MF+1} \boldsymbol{TSS}(\boldsymbol{k}). \quad \textbf{(19)}$$

Because the individual test statistic $\boldsymbol{TSS}(\boldsymbol{k})$ is asymptotically independent and has a chi-square distribution with one degree of freedom for each k, the AH test statistic will have a chi-square distribution with $MF+1$ degrees of freedom.

It is demonstrated by Wild (2001) that the AH test has reasonable size properties. Specifically, the size results were generally conservative for small N, M, and $MF+1$ and tend to converge toward their nominal levels as the sample size is increased after typically becoming slightly more conservative in moderate samples.

The issue of spectral shape was found to be important only in small samples and made little difference to the size results obtained in moderate and large samples.

When the number of estimated frequencies $MF + 1$ was increased substantially, for example, from 11 to 61, problems emerged. Specifically, the AH test statistic tended to significantly overreject in small samples. This tends to correct itself in moderate and large samples. Furthermore, the power tests documented by Wild (2001) successfully demonstrate the AH test's capabilities in detecting shifts in variance and autocovariance structure.

The AH test statistic's applicability to a real-world macroeconomic problem is demonstrated by applying the test to the Australian Building Society Deposits (ABSD) example investigated by Foster and Wild (1999a).

4. THE AUSTRALIAN BUILDING SOCIETY DEPOSIT (ABSD) EXAMPLE REVISITED

In this section, we apply the AH test to the ABSD example investigated by Foster and Wild (1999a). This provides a very clear example of a logistic-diffusion process with emergent self-organizational features culminating in a phase of saturation in the early 1980's with inherent instabilities [see Foster and Wild (1999a, pp. 118, 121)]. The data we use in this section are the set of residuals obtained from the ABSD augmented logistic regression equation outlined by Foster and Wild (1999a) for the sample period November 1967 to September 1988. These residuals were calculated using the Pc-Give 8 econometric package [Doornik and Hendry (1994)] by computing the one-step residuals from the estimated relationship and then standardizing by dividing each residual by the standard error of the estimated regression equation. The time path of these residuals is documented in Figure 4.

In the results reported below, we use the coherence to measure dependence between frames and adopt the following parameter settings: the frame size $(N) = 100$, number of estimated frequencies $(MF + 1) = 11$, and truncation parameter $(M) = 10$.

The descriptive statistics of the original set of residuals from the ABSD logistic regression equation are listed in Table 1. Clearly, the mean of the process is zero. Note further that the set of residuals does not "trip" the normality test. This test is conservative in small samples as simulations have suggested a bias toward the rejection of the null hypothesis of normality [Doornik and Hendry (1994)]. Therefore, if we cannot reject the null, then this result is "believable" with regard to the falsification of the null hypothesis.

The coherence is used to measure dependence between frames. The coherence can be viewed as measuring the correlation between the two frames as a function of frequency and can take values between 0 and 1. It has the same interpretation as given conventionally to the coefficient of multiple correlation. The coherence between the first and last frames is documented in Table 2.

The low coherence values listed in Table 2 would indicate that the two frames can be regarded as being approximately independent. This can be confirmed formally

TABLE 1. Descriptive statistics of original source series

Statistic	Value
Mean	−0.126E-10
Std dev	0.963
Skew	−0.205
Kurtosis	0.242
$C(6)$	−2.701
Max value	2.51
Min value	−2.95
Fourth-order moment	2.79
Fourth-order cumulant	0.208
P-value for the Bowman and Shenton normality test	0.307

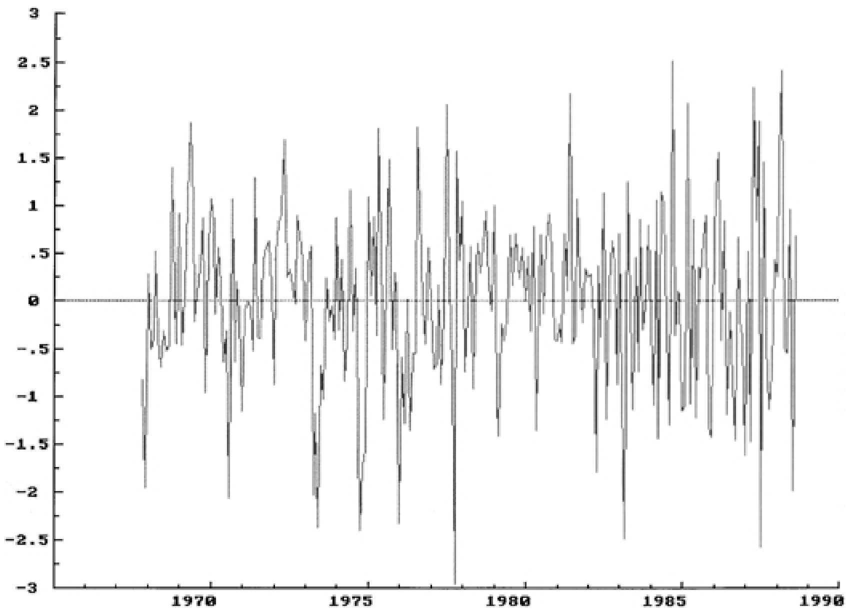

FIGURE 4. Time path of standardized residuals, November 1967–September 1988.

by employing a test to determine the cut-off value for the coherence for a particular frequency, above which the null hypothesis of zero coherence (independence) can be rejected, signifying dependence between the two frames at that particular frequency. The test statistic has the following form and distribution:

$$\frac{2M|\hat{w}_{ij}(k)|}{(1-|\hat{w}_{ij}(k)|)} = F_{2,4M}, \tag{20}$$

TABLE 2. Coherence for frames 1 and 151

Frequency	Coherence
0	0.002
1	0.015
2	0.021
3	0.026
4	0.015
5	0.003
6	0.010
7	0.021
8	0.009
9	0.068
10	0.081

where $\hat{w}_{ij}(k)$ is the estimated coherence between frames i and j at frequency k [consult Fuller (1976, pp. 315, 319–320), and Priestley (1981, p. 706)].

The critical value for $\hat{w}_{ij}(k)$ can be determined from (20) once we substitute in the critical value of the F distribution, given that $M = 10$. Choosing the 5% and 1% levels of significance for α, and associated critical values for the F distribution,[13] produces the following critical values for $\hat{w}_{ij}(k)$:

$$\hat{w}_{ij}(k) \le 0.128, \quad \text{for } \alpha = 0.05, \tag{21a}$$

and

$$\hat{w}_{ij}(k) \le 0.190, \quad \text{for } \alpha = 0.01. \tag{21b}$$

We reject the null hypothesis if $\hat{w}_{ij}(k) > 0.128$ or 0.190, at the 5% and 1% levels of significance, respectively, and conclude that there is dependence between the two frames at frequency k. In the ABSD case study being considered, it is evident from the size of the estimated coherence between frames 1 and 151 listed in Table 2, that we cannot reject the null hypothesis at any of the estimated frequencies.[14]

The classical variance values associated with the $\boldsymbol{TS}(\boldsymbol{k})$ statistic are documented in Table 3.

The AH test statistic result is documented in Table 4. Recall that this test is a joint test of the null hypothesis of homogeneity at all estimated frequencies. The aggregate degrees of freedom $(MF + 1)$ are 11. Inspection of Table 4 indicates strong rejection of the null hypothesis of homogeneity at the 0.05% level of significance as indicated by the P-value of 0.0000 listed in Table 4.

The size results associated with the AH statistic reported by Wild (2001) indicate a propensity for slight overrejection for the parameter values adopted in this section for N, M, and $MF + 1$; consult col. A, Experiment 1, Table 1, of Wild (2001). However, the actual empirical size level computed for the 0.05%

Table 3. Variance of $\boldsymbol{TS(k)}$ statistic

Frequency	Classical variance
0	0.2153
1	0.1062
2	0.1056
3	0.1051
4	0.1063
5	0.1075
6	0.1067
7	0.1056
8	0.1069
9	0.1006
10	0.1983

Table 4. Homogeneity test result: AH test statistic

Original series, classical variance	
Statistic	Value
$\chi^2(11)$	43.487
P-value	0.0000

level of significance generates an associated critical P-value of $(1.0000000 - 0.9997532) = 0.000468$. The P-value listed in Table 4 is clearly smaller than this critical value, thereby giving additional support for the conclusions made above about the nature of the rejections secured by the AH test statistic. These results also rigorously verify statistically the conjecture made by Foster and Wild (1999a) concerning the existence of heterogeneity and evolutionary change associated with the qualitative change in spectral decomposition between the two frames.

5. CONCLUSIONS

A Cusum-based joint homogeneity test, termed the aggregate homogeneity (AH) test statistic, is based on assessment of uniformity in spectral decomposition of residuals from estimated econometric relationships. The actual concept of heterogeneity developed in this paper encompasses shifts in both variance and autocovariance structure. These forms of heterogeneity are viewed as operating, fundamentally, by changing the *magnitude* and *shape* of power spectra.

Under the null hypothesis of homogeneity, the distribution theory developed is applicable to the class of real stationary processes with absolutely summable autocovariance and fourth-order cumulant structure. The test statistic seeks to determine whether the spectral decomposition associated with one frame of the residuals is the same as the spectral decomposition of another frame, in a statistical sense. This means that any variation in spectral decomposition between the two frames should be able to be attributed to random chance. If the variation is greater than what can be reasonably attributed to random chance, then evidence of heterogeneity and nonstationarity in the second-order moment structure of the process will be forthcoming, providing direct evidence of evolutionary change.

The AH test utilizes the asymptotic independence and normality of logarithmically transformed nonnormalized spectral density estimates to construct a Cusum test statistic.

As an example, the AH test statistic was applied to the residuals from the Australian Building Society Deposits (ABSD) regression equation documented by Foster and Wild (1999a). The results from applying the test indicate that the second-order (variance/autocovariance) structure of the residuals is heterogeneous, confirming the existence of weak nonstationarity and evolutionary change, thereby rigorously confirming statistically the qualitative results previously documented by Foster and Wild (1999a).

NOTES

1. The extension to multivariate problems is straightforward but nontrivial in terms of the additional notation and spectral analysis concepts that must be introduced.

2. In this context, the expression "nonlinearity in functional form" could refer to nonlinearity in variables and/or parameters.

3. This assumption is taken to mean that the deterministic part of the process has a constant parametric structure. This would rule out structural change associated with parameter variation. This assumption is necessary if (1a) is to have meaningful equilibrium properties.

4. A process has a time-reversible probabilistic structure if the probabilistic structure of the time series going forward in time is identical to that in reverse time.

5. These schemes admit transfer functions which are time invariant and linear in parameters.

6. Here, the term "descriptive" refers to the way rolling regression techniques have been activated in econometric packages. Specifically, wide use of plots of estimated coefficients from rolling window regressions are employed instead of formal hypothesis tests.

7. A frame of data is defined as a subgrouping of successive data points taken from the same data series $x(t),\ t = 1, \dots, T$. The data series generally will be a set of residuals obtained from a regression model. Typically, the frame length is much smaller than the sample size T.

8. However, parameter $MF + 1$, which denotes the number of individual frequencies to be estimated, determines the number of grid points in the interval $(0, \pi)$, which determines the magnitude of the difference between neighboring frequencies. For example, if $MF + 1 = 10$ and 100, then the grid sizes are 0.31416 and 0.031416, respectively. Therefore, the larger the variable $MF + 1$, the smaller is the absolute value of the difference between neighboring frequencies. This indicates that neighboring frequencies are more likely to be uncorrelated if the window truncation parameter M is large relative to parameter $MF + 1$. This observation is supported by results from the size simulations outlined by Wild (2001).

9. For this result to hold identically, the two frames must have identical probability distributions. If the frames have identical spectra but not identical distributions, then equations (9) and (10) hold only asymptotically.

10. Equation (8) could be amended for bias associated with other types of lag windows by including the appropriate expression for asymptotic bias outlined in Table 6.1 of Priestley (1981, p. 463).

11. Because the Parzen lag window is used in spectral estimation, $K = 0.539285$ [see Priestley (1981, p. 463)].

12. The coherence expression $[\boldsymbol{c}^2(i, j)k + \boldsymbol{q}^2(i, j)k]/\boldsymbol{f}_k^2$ has been termed the squared coherence by some authors: consult Priestley (1981, p. 661). In this paper, we associate the term coherence with the above expression. Also consult Jenkins (1961, 1963).

13. See, for example, Johnston (1972, pp. 428–429).

14. This test only addresses the issue of dependence at the second-order level. It is possible, however, that the frames might be dependent at higher-order levels. As such, this test is not a complete test of independence unless the process generating the residuals is Gaussian. However, note also that the result of the Bowman–Shenton normality test documented with the descriptive statistics indicates that the residuals are Gaussian.

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APPENDIX: ESTIMATION OF POWER SPECTRA AND COHERENCE

It is assumed that we want to estimate $MF+1$ individual frequencies, comprising the frequency set $\{0, 1, \ldots, MF\}$. We employ the Parzen window in spectral estimation. The power spectrum can be estimated using the following formula:

$$\hat{f}(k) = \hat{f}\left(\frac{\pi l}{MF}\right) = \frac{1}{2\pi}\left\{\hat{C}(0) + 2\sum_{s=1}^{M/2}\left[1 - 6\left(\frac{s}{M}\right)^2 + 6\left(\frac{s}{M}\right)^3\right] * \hat{C}(s) * \cos\left(\frac{\pi l s}{MF}\right) + 4\sum_{s=M/2+1}^{M}\left[\left(1-\frac{s}{M}\right)^3\right] * \hat{C}(s) * \cos\left(\frac{\pi l s}{MF}\right)\right\}, \tag{A.1}$$

where $l = 0, \ldots, MF$ and $s = 1, \ldots, M$, where M is the integer value adopted for the window truncation parameter.

The term $\hat{C}(s)$ denotes the estimate of the autocovariance function at lag s. The autocovariance function is estimated from the following equation:

$$\hat{C}(s) = \frac{1}{N}\sum_{t=1}^{N-s}(X_t - X)(X_{t+s} - X), \quad s = 0, 1, \ldots, M, \tag{A.2}$$

where N is the frame length and the mean of X_t, denoted $\bar{X}$, is estimated from

$$\bar{X} = \frac{1}{N}\sum_{t=1}^{N} X_t. \tag{A.3}$$

The cospectral and quadrature spectral estimates are obtained from the following formulas where, for notational purposes, we associate with frame i the series $\{X_t\}$ and with frame j the series $\{Y_t\}$:

$$\begin{aligned}\hat{c}(i, j)k = \hat{c}_{ij}\left(\frac{\pi l}{MF}\right) &= \frac{1}{2\pi}\left\{\hat{C}_{ij}(0) + \sum_{s=1}^{M/2}\left[1 - 6\left(\frac{s}{M}\right)^2 + 6\left(\frac{s}{M}\right)^3\right]\right.\\ &* [\hat{C}_{ij}(s) + \hat{C}_{ji}(s)] * \cos\left(\frac{\pi ls}{MF}\right) + 2\sum_{s=M/2+1}^{M}\left[\left(1 - \frac{s}{M}\right)^3\right]\\ &\left.* [\hat{C}_{ij}(s) + \hat{C}_{ji}(s)] * \cos\left(\frac{\pi ls}{MF}\right)\right\},\end{aligned} \tag{A.4}$$

$$\begin{aligned}\hat{q}(i, j)k = \hat{q}_{ij}\left(\frac{\pi l}{MF}\right) &= \frac{1}{2\pi}\left\{\sum_{s=1}^{M/2}\left[1 - 6\left(\frac{s}{M}\right)^2 + 6\left(\frac{s}{M}\right)^3\right] * [\hat{C}_{ij}(s) - \hat{C}_{ji}(s)]\right.\\ &\left.* \sin\left(\frac{\pi ls}{MF}\right) + 2\sum_{s=M/2+1}^{M}\left[\left(1 - \frac{s}{M}\right)^3\right] * [\hat{C}_{ij}(s) - \hat{C}_{ji}(s)] * \sin\left(\frac{\pi ls}{MF}\right)\right\}.\end{aligned} \tag{A.5}$$

Given (A.4) and (A.5), the coherence is estimated from the following equation:

$$\hat{\boldsymbol{R}}^2(\boldsymbol{i},\boldsymbol{j})\boldsymbol{k} = \hat{C}_r\left(\frac{\pi l}{MF}\right)^2 = \frac{\left[\hat{c}_{ij}\left(\frac{\pi l}{MF}\right)\right]^2 + \left[\hat{q}_{ij}\left(\frac{\pi l}{MF}\right)\right]^2}{\hat{f}_i\left(\frac{\pi l}{MF}\right) * \hat{f}_j\left(\frac{\pi l}{MF}\right)} \tag{A.6}$$

and from (A.4) and (A.5),

$$\hat{C}_{ij}(s) = \frac{1}{N}\sum_{t=1}^{N-s}(X_t - X)(Y_{t+s} - Y), \qquad s = 0, 1, \ldots, M, \tag{A.7}$$

where

$$\bar{X} = \frac{1}{N}\sum_{t=1}^{N} X_t \quad \text{and} \quad \bar{Y} = \frac{1}{N}\sum_{t=1}^{N} Y_t. \tag{A.8}$$